The Analytic Hierarchy Process (AHP) for Decision Making By Thomas Saaty

Decision Making involves setting priorities and the AHP is the methodology for doing that.

Real Life Problems Exhibit:

Strong Pressures and Weakened Resources

Complex Issues - Sometimes There are No "Right" Answers

Vested Interests

Conflicting Values

Most Decision Problems are Multicriteria

- Maximize profits
- Satisfy customer demands
- Maximize employee satisfaction
- Satisfy shareholders
- Minimize costs of production
- Satisfy government regulations
- Minimize taxes
- Maximize bonuses

Decision Making

Decision making today is a science. People have hard decisions to make and they need help because many lives may be involved, the survival of the business depends on making the right decision, or because future success and diversification must survive competition and surprises presented by the future.

WHAT KIND AND WHAT AMOUNT OF KNOWLEDGE TO MAKE DECISIONS

Some people say

- What is the use of learning about decision making? Life is so complicated that the factors which go into a decision are beyond our ability to identify and use them effectively.

I say that is not true.

- We have had considerable experience in the past thirty years to structure and prioritize thousands of decisions in all walks of life. We no longer think that there is a mystery to making good decisions.

THE GOODS THE BADS AND THE INTANGIBLES

- Decision Making involves all kinds of tradeoffs among intangibles. To make careful tradeoffs we need to measure things because a bad may be much more intense than a good and the problem is not simply exchanging one for the other but they must be measured quantitatively and swapped.
- One of the major problems that we have had to solve has been how to evaluate a decision based on its benefits, costs, opportunities, and risks. We deal with each of these four merits separately and then combine them for the overall decision.

3 Kinds of Decisions

a) Instantaneous and personal like what restaurant to eat at and what kind of rice cereal to buy; b) Personal but allowing a little time like which job to choose and what house to buy or car to drive; c) Long term decisions and any decisions that involve planning and resource allocation and more significantly group decision making.

We can use the AHP and ANP as they are. Personal decisions need to be automated with data and judgments by different types of people so every individual can identify with one of these groups whose judgments for the criteria he would use and which uses the rating approach for all the possible alternatives in the world so one can quickly choose what he prefers after identifying with that type of people. A chip needs to be installed for this purpose for example in a cellular phone.

Knowledge is Not in the Numbers

Isabel Garuti is an environmental researcher whose father-in-law is a master chef in Santiago, Chile. He owns a well known Italian restaurant called Valerio. He is recognized as the best cook in Santiago. Isabel had eaten a favorite dish risotto ai funghi, rice with mushrooms, many times and loved it so much that she wanted to learn to cook it herself for her husband, Valerio's son, Claudio. So she armed herself with a pencil and paper, went to the restaurant and begged Valerio to spell out the details of the recipe in an easy way for her. He said it was very easy. When he revealed how much was needed for each ingredient, he said you use a little of this and a handful of that. When it is O.K. it is O.K. and it smells good. No matter how hard she tried to translate his comments to numbers, neither she nor he could do it. She could not replicate his dish. Valerio knew what he knew well. It was registered in his mind, this could not be written down and communicated to someone else. An unintelligent observer would claim that he did not know how to cook, because if he did, he should be able to communicate it to others. But he could and is one of the best.

Valerio <u>can</u> say, "Put more of this than of that, don't stir so much," and so on. That is how he cooks his meals - by following his instincts, not formalized logically and precisely.
The question is: How does he synthesize what he knows?

Knowing Less, Understanding More

You don't need to know everything to get to the answer.

Expert after expert missed the revolutionary significance of what Darwin had collected. Darwin, who knew less, somehow understood more.

Aren't Numbers Numbers? We have the habit to crunch numbers whatever they are

An elderly couple looking for a town to which they might retire found Summerland, in Santa Barbara County, California, where a sign post read:

Summerland

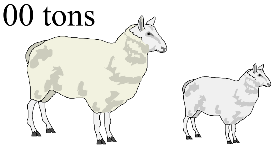
Population	3001
Feet Above Sea Level	208
Year Established	1870
Total	5079

"Let's settle here where there is a sense of humor," said the wife; and they did.

Do Numbers Have an Objective Meaning?

Butter: 1, 2,…, 10 lbs.; 1,2,…, 100 tons

Sheep: 2 sheep (1 big, 1 little)



Temperature: 30 degrees Fahrenheit to New Yorker, Kenyan, Eskimo

Since we deal with **Non-Unique Scales** such as [lbs., kgs], [yds, meters], [Fahr., Celsius] and such scales cannot be combined, we need the idea of **PRIORITY**.

PRIORITY becomes an abstract unit valid across all scales.

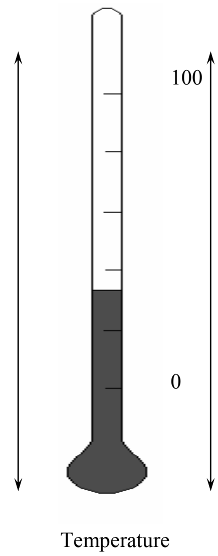
A priority scale based on preference is the AHP way to standardize non-unique scales in order to combine multiple criteria.

Nonmonotonic Relative Nature of Absolute Scales

Good for
preserving food

Bad for
preserving food

Good for
preserving food



Bad for
comfort

Good for
comfort

Bad for
comfort

OBJECTIVITY!?

Bias in upbring: objectivity is agreed upon subjectivity. We interpret and shape the world in our own image. We pass it along as fact. In the end it is all obsoleted by the next generation.

Logic breaks down: Russell-Whitehead Principia; Gödel's Undecidability Proof.

Intuition breaks down: circle around earth; milk and coffee.

How do we manage?

Making a Decision

Widget B is cheaper than Widget A

Widget A is better than Widget B

Which Widget would you choose?

Basic Decision Problem

Criteria: Low Cost > Operating Cost > Style

Car:	A	B	B
	V	V	V
Alternatives:	B	A	A

Suppose the criteria are preferred in the order shown and the cars are preferred as shown for each criterion. Which car should be chosen? It is desirable to know the strengths of preferences for tradeoffs.

To understand the world we assume that:

We can describe it

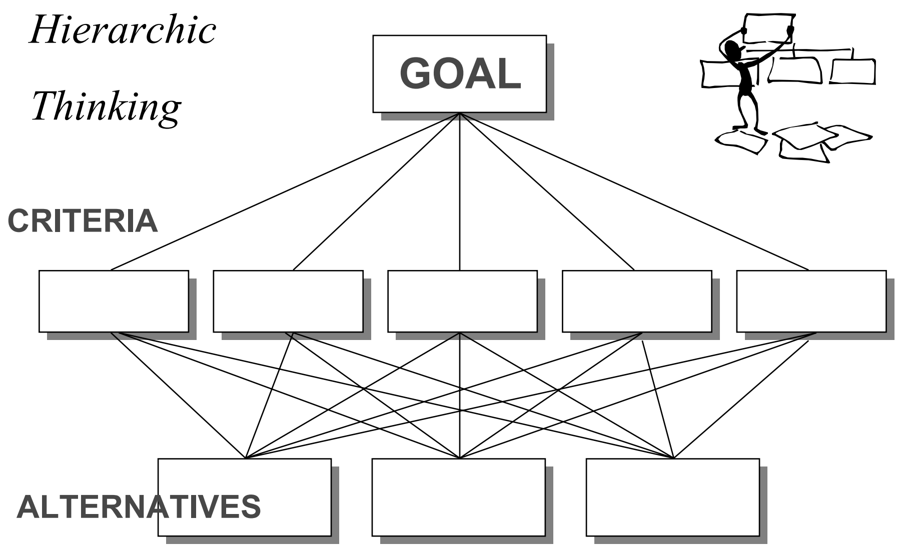
We can define relations between its parts and

We can apply judgment to relate the parts according to

a goal or purpose that we have in mind.

Hierarchic

Thinking



Power of Hierarchic Thinking

A hierarchy is an efficient way to organize complex systems. It is efficient both structurally, for represent-ing a system, and functionally, for controlling and passing information down the system.

Unstructured problems are best grappled with in the systematic framework of a hierarchy or a feedback network.

Order, Proportionality and Ratio Scales

- All order, whether in the physical world or in human thinking, involves proportionality among the parts, establishing harmony and synchrony among them. Proportionality means that there is a ratio relation among the parts. Thus, to study order or to create order, we must use ratio scales to capture and synthesize the relations inherent in that order. The question is how?

Relative Measurement
The Process of Prioritization

In relative measurement a preference, judgment is expressed on each pair of elements with respect to a common property they share.

In practice this means that a pair of elements in a level of the hierarchy are compared with respect to parent elements to which they relate in the level above.

Relative Measurement (cont.)

If, for example, we are comparing two apples according to weight we ask:

- Which apple is bigger?
- How much bigger is the larger than the smaller apple?
  Use the smaller as the unit and estimate how many more times bigger is the larger one.
- The apples must be relatively close (homogeneous) if we hope to make an accurate estimate.

Relative Measurement (cont.)

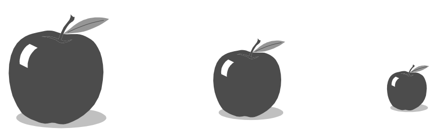
•The Smaller apple then has the reciprocal value when compared with the larger one. There is no way to escape this sort of reciprocal comparison when developing judgments

•If the elements being compared are not all homogeneous, they are placed into homogeneous groups of gradually increasing relative sizes (homogeneous clusters of homogeneous elements).

• Judgments are made on the elements in one group of small elements, and a "pivot" element is borrowed and placed in the next larger group and its elements are compared. This use of pivot elements enables one to successively merge the measurements of all the elements. Thus homogeneity serves to enhance the accuracy of measurement.

Comparison Matrix

Given: Three apples of different sizes.

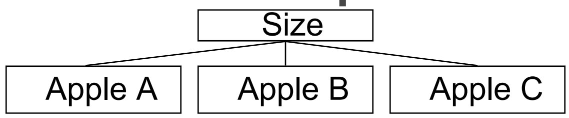


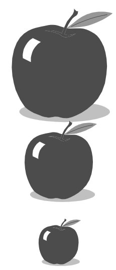
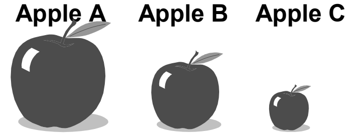
Apple A Apple B Apple C

We Assess Their Relative Sizes By Forming Ratios

Size Comparison	Apple A	Apple B	Apple C
Apple A	S_1/S_1	S_1/S_2	S_1/S_3
Apple B	S_2 / S_1	S_2 / S_2	S_2 / S_3
Apple C	S_3 / S_1	S_3 / S_2	S_3 / S_3

Pairwise Comparisons



Size Comparison	Apple A	Apple B	Apple C	Resulting Priority Eigenvector	Relative Size of Apple
					
Apple A	**1**	**2**	**6**	**6/10**	**A**
Apple B	**1/2**	**1**	**3**	**3/10**	**B**
Apple C	**1/6**	**1/3**	**1**	**1/10**	**C**

When the judgments are consistent, as they are here, any normalized column gives the priorities.

Pairwise Comparisons using Judgments and the Derived Priorities

Nicer ambience comparisons	Paris	London	New York	**Normalized**	**Total**
					
Paris	1	2	5	0.5815	1
London	1/2	1	3	0.3090	0.5328
New York	1/5	1/3	1	0.1095	0.1888

Pairwise Comparisons using Judgments and the Derived Priorities

Politician comparisons	**B. Clinton** 	**M. Tatcher** 	**G. Bush** 	Normalized	Total
 **B. Clinton**	1	3	7	0.6220	1
 **M. Tatcher**	1/3	1	5	0.2673	0.4297
 **G. Bush**	1/7	1/5	1	0.1107	0.1780

SCORING AND PAIRED COMPARISONS

In *scoring* one guesses at numbers to assign to things and when one normalizes, everything falls between zero and one and can look respectable because if we know the ordinal ranking of things, then assigning them comparable numbers yields decimals that have the appropriate order and also differ by a little from each other and lie between zero and one, it sounds fantastic despite guessing at the numbers.

Paired comparisons is a scientific process in which the smaller or lesser element serves as the unit and the larger or greater one is estimated as a multiple of that unit. Although one can say that here too we have guessing but it is very different because we know what we are supposed to do and not just pull a number out of a hat. Therefore one would expect better answers from paired comparisons. If the person making the comparisons knows nothing about the elements being compared, his outcome would be just as poor as the other. But if he does know the elements well, one would expect very good results.

When the judgments are consistent, we have two ways to get the answer:

1. By adding any column and dividing each entry by the total, that is by **normalizing** the column, any column gives the same result. A quick test of consistency if all the columns give the same answer.
2. By adding the rows and normalizing the result.

When the judgments are inconsistent we have two ways to get the answer:

1. An approximate way: By normalizing each column, forming the row sums and then normalizing the result.
2. The exact way: By raising the matrix to powers and normalizing its row sums

Consistency

In this example Apple B is 3 times larger than Apple C. We can obtain this value directly from the comparisons of Apple A with Apples B & C as 6/2 = 3. But if we were to use judgment we may have guessed it as 4. In that case we would have been inconsistent.

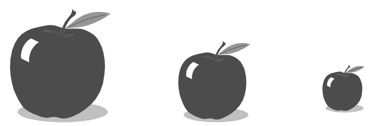
Now guessing it as 4 is not as bad as guessing it as 5 or more. The farther we are from the true value the more inconsistent we are. The AHP provides a theory for checking the inconsistency throughout the matrix and *allowing a certain level of overall inconsistency but not more.*

Consistency (cont.)

- Consistency itself is a necessary condition for a better understanding of relations in the world but it is not sufficient. For example we could judge all three of the apples to be the same size and we would be perfectly consistent, but very wrong.

- We also need to improve our validity by using redundant information.

- It is fortunate that the mind is not programmed to be always consistent. Otherwise, it could not integrate new information by changing old relations.



Consistency (cont.)

Because the world of experience is vast and we deal with it in pieces according to whatever goals concern us at the time, our judgments can never be perfectly precise.

It may be impossible to make a consistent set of judgments on some pieces that make them fit exactly with another consistent set of judgments on other related pieces. So we may neither be able to be perfectly consistent nor want to be.

We must allow for a modicum of inconsistency. This explanation is the basis of fuzziness in knowledge. To capture this kind of fuzziness one needs ratio scales.

Fuzzy Sets have accurately identified the nature of inconsistency in measurement but has not made the link with ratio scales to make that measurement even more precise and grounded in a sound unified theory of ratio scales. Fuzzy Sets needs the AHP!

Consistency (cont.)

How Much Inconsistency to Tolerate?

- Inconsistency arises from the need for redundancy.
- Redundancy improves the validity of the information about the real world.
- Inconsistency is important for modifying our consistent understanding, but it must not be too large to make information seem chaotic.
- Yet inconsistency cannot be negligible; otherwise, we would be like robots unable to change our minds.
- Mathematically the measurement of consistency should allow for inconsistency of no more than one order of magnitude smaller than consistency. Thus, an inconsistency of no more than 10% can be tolerated.
- This would allow variations in the measurement of the elements being compared without destroying their identity.
- As a result the number of elements compared must be small, i.e. seven plus or minus two. Being homogeneous they would then each receive about ten to 15 percent of the total relative value in the vector of priorities.
- A small inconsistency would change that value by a small amount and their true relative value would still be sufficiently large to preserve that value.
- Note that if the number of elements in a comparison is large, for example 100, each would receive a 1% relative value and the small inconsistency of 1% in its measurement would change its value to 2% which is far from its true value of 1%.

Comparison of Intangibles

The same procedure as we use for size can be used to compare things with intangible properties. For example, we could also compare the apples for:

- TASTE
- AROMA
- RIPENESS

The Analytic Hierarchy Process (AHP) is the Method of Prioritization

- AHP captures priorities from paired comparison judgments of the
- elements of the decision with respect to each of their parent criteria

- Paired comparison judgments can be arranged in a matrix.

- Priorities are derived from the matrix as its principal eigenvector,
- which defines a ratio scale. Thus, the eigenvector is an intrinsic
- concept of a correct prioritization process. It also allows for the
- measurement of inconsistency in judgment.

- Priorities derived this way satisfy the property of a ratio scale
- just like pounds and yards do.

Decision Making

We need to prioritize both tangible and intangible criteria:

- ♦ In most decisions, intangibles such as
  - political factors and
  - social factors

  take precedence over tangibles such as
  - economic factors and
  - technical factors
- ♦ It is not the precision of measurement on a particular factor that determines the validity of a decision, but the importance we attach to the factors involved.
- ♦ How do we assign importance to all the factors and synthesize this diverse information to make the best decision?

Verbal Expressions for Making Pairwise Comparison Judgments

Equal importance

Moderate importance of one over another

Strong or essential importance

Very strong or demonstrated importance

Extreme importance

Fundamental Scale of Absolute Numbers Corresponding to Verbal Comparisons

1	Equal importance
3	Moderate importance of one over another
5	Strong or essential importance
7	Very strong or demonstrated importance
9	Extreme importance
2,4,6,8	Intermediate values
	Use Reciprocals for Inverse Comparisons

Which Drink is Consumed More in the U.S.?

An Example of Estimation Using Judgments

Drink Consumption in the U.S.	Coffee	Wine	Tea	Beer	Sodas	Milk	Water
Coffee	1	9	5	2	1	1	1/2
Wine	1/9	1	1/3	1/9	1/9	1/9	1/9
Tea	1/5	2	1	1/3	1/4	1/3	1/9
Beer	1/2	9	3	1	1/2	1	1/3
Sodas	1	9	4	2	1	2	1/2
Milk	1	9	3	1	1/2	1	1/3
Water	2	9	9	3	2	3	1

The derived scale based on the judgments in the matrix is:

Coffee	Wine	Tea	Beer	Sodas	Milk	Water
.177	.019	.042	.116	.190	.129	.327

with a consistency ratio of .022.

The actual consumption (from statistical sources) is:

.180	.010	.040	.120	.180	.140	.330

Estimating which Food has more Protein

Food Consumption in the U.S.	A	B	C	D	E	F	G
A: Steak	1	9	9	6	4	5	1
B: Potatoes		1	1	1/2	1/4	1/3	1/4
C: Apples			1	1/3	1/3	1/5	1/9
D: Soybean				1	1/2	1	1/6
E: Whole Wheat Bread		(Reciprocals)			1	3	1/3
F: Tasty Cake						1	1/5
G: Fish							1

The resulting derived scale and the actual values are shown below:

	Steak	Potatoes	Apples	Soybean	W. Bread	T. Cake	Fish
Derived	.345	.031	.030	.065	.124	.078	.328
Actual	.370	.040	.000	.070	.110	.090	.320

(Derived scale has a consistency ratio of .028.)

WEIGHT COMPARISONS

Weight	Radio	Typewriter	Large Attache Case	Projector	Small Attache	Eigenvector	Actual Relative Weights
Radio	1	1/5	1/3	1/4	4	0.09	0.10
Typewriter	5	1	2	2	8	0.40	0.39
Large Attache Case	3	1/2	1	1/2	4	0.18	0.20
Projector	4	1/2	2	1	7	0.29	0.27
Small Attache Case	1/4	1/8	1/4	1/7	1	0.04	0.04

DISTANCE COMPARISONS

Comparison of Distances from Philadelphia	*Cairo*	*Tokyo*	*Chicago*	*San Francisco*	*London*	*Montreal*	*Eigen-vector*	*Distance to Philadelphia in miles*	*Relative Distance*
Cairo	1	1/2	8	3	3	7	0.263	5,729	0.278
Tokyo	3	1	9	3	3	9	0.397	7,449	0.361
Chicago	1/8	1/9	1	1/6	1/5	2	0.033	660	0.032
San Francisco	1/3	1/3	6	1	1/3	6	0.116	2,732	0.132
London	1/3	1/3	5	3	1	6	0.164	3,658	0.177
Montreal	1/7	1/9	1/2	1/6	1/6	1	0.027	400	0.019

Perimeter Problem

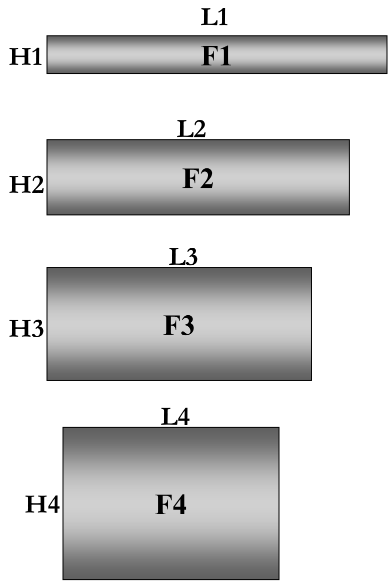


Fig.1 : $L_1 = 9$, $H_1 = 1$
P = 20

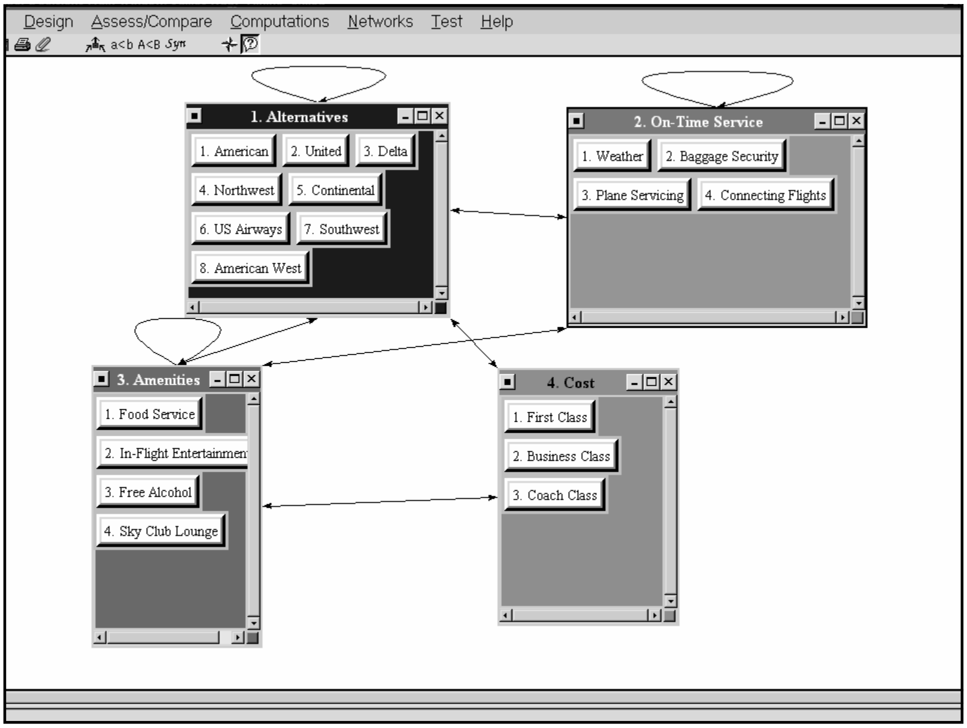
Fig.2: $L_2 = 8$, $H_2 = 2$
P = 20

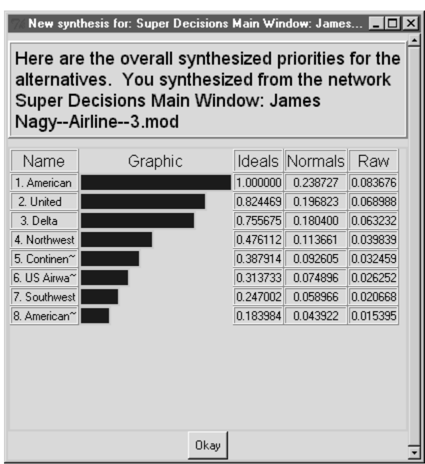
Fig.3 : $L_3 = 7$, $H_3 = 3$
P = 20

Fig.4 : $L_4 = 6$, $H_4 = 4$
P = 20

All Four Figures have the same Perimeter

	Length	Width	Perimeter	Relative
F1	9	1	20	.25
F2	8	2	20	.25
F3	7	3	20	.25
F4	6	4	20	.25




New synthesis for: Super Decisions Main Window: James...

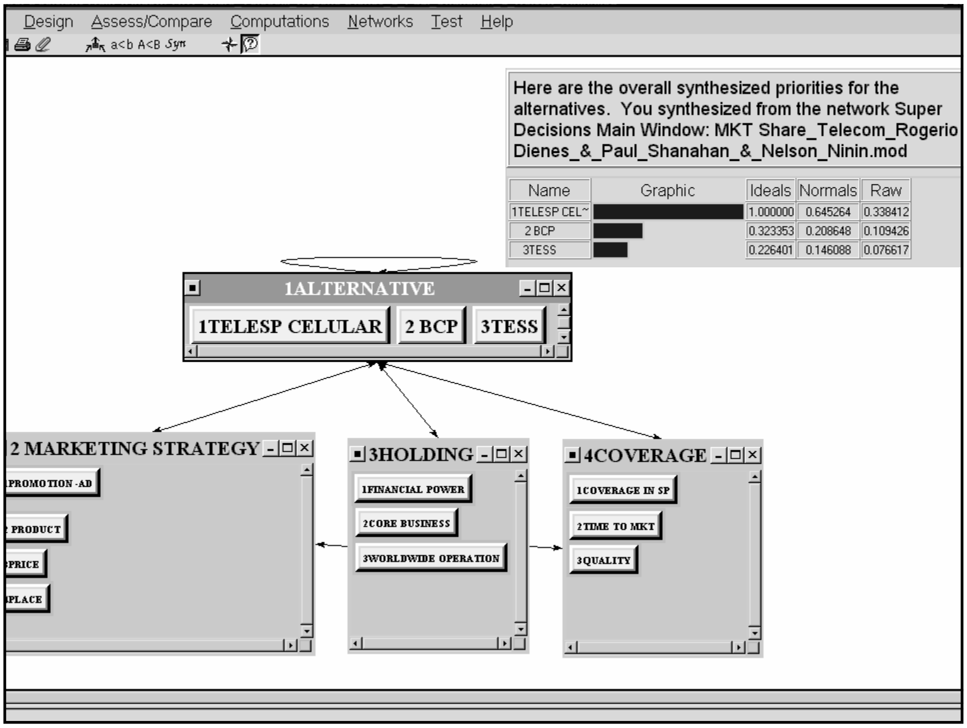
Here are the overall synthesized priorities for the alternatives. You synthesized from the network Super Decisions Main Window: James Nagy--Airline--3.mod

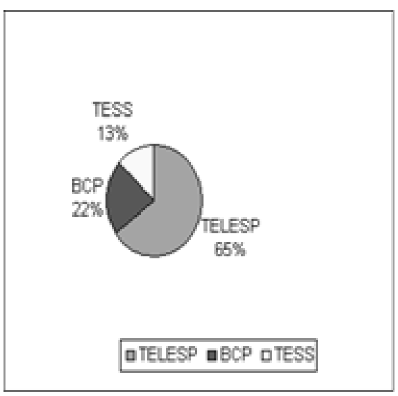
Name	Graphic	Ideals	Normals	Raw
1. American		1.000000	0.238727	0.083676
2. United		0.824469	0.196823	0.068988
3. Delta		0.755675	0.180400	0.063232
4. Northwest		0.476112	0.113661	0.039839
5. Continen~		0.387914	0.092605	0.032459
6. US Airwa~		0.313733	0.074896	0.026252
7. Southwest		0.247002	0.058966	0.020668
8. American~		0.183984	0.043922	0.015395

Okay

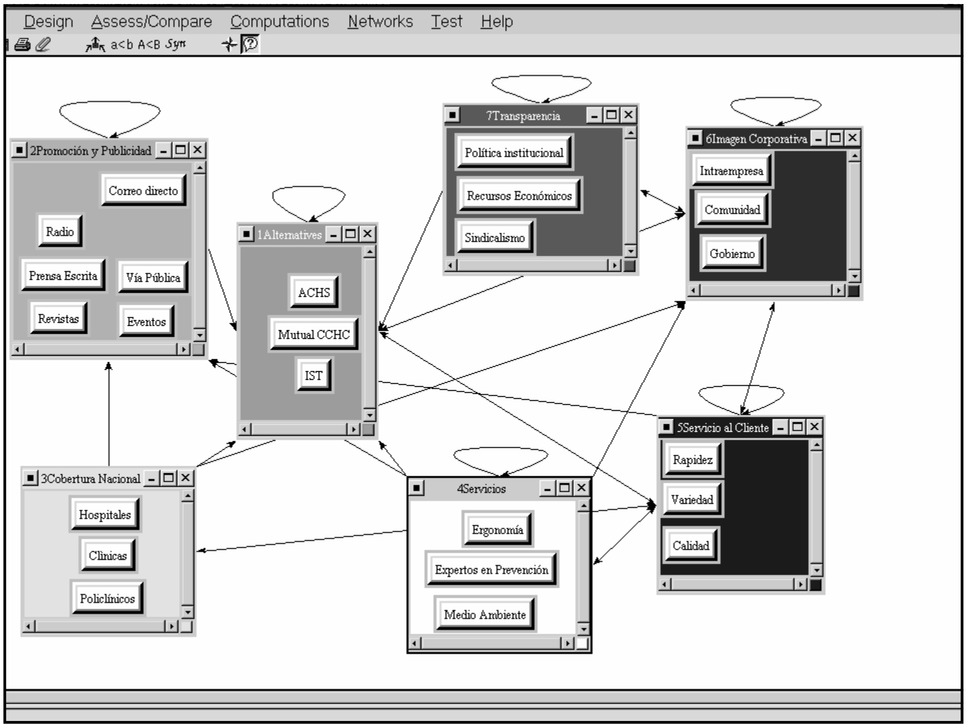
Nagy Airline Market Share Model

	Model	Actual (Yr 2000)
American	23.9	24.0
United	18.7	19.7
Delta	18.0	18.0
Northwest	11.4	12.4
Continental	9.3	10.0
US Airways	7.5	7.1
Southwest	5.9	6.4
Amer.West	4.4	2.9





	Income	Relative Share (Income)	Relative Share (Model)
Total	7,914,051		
TELESP	5,104,000	64.5	64.5
BCP	1,778,951	22.5	20.9
TESS	1,032,000	13.0	14.6



Comparación Modelo ANP v/s Realidad Actual.

	ANP Results	Actual Results
Asociación Chilena de Seguridad (ACHS)	52,0 %	52,6 %
Mutual de Seguridad	35,5 %	34,8 %
Instituto Seguros del Trabajo (IST)	12,5 %	12,6 %
Total	100,0 %	100,0 %

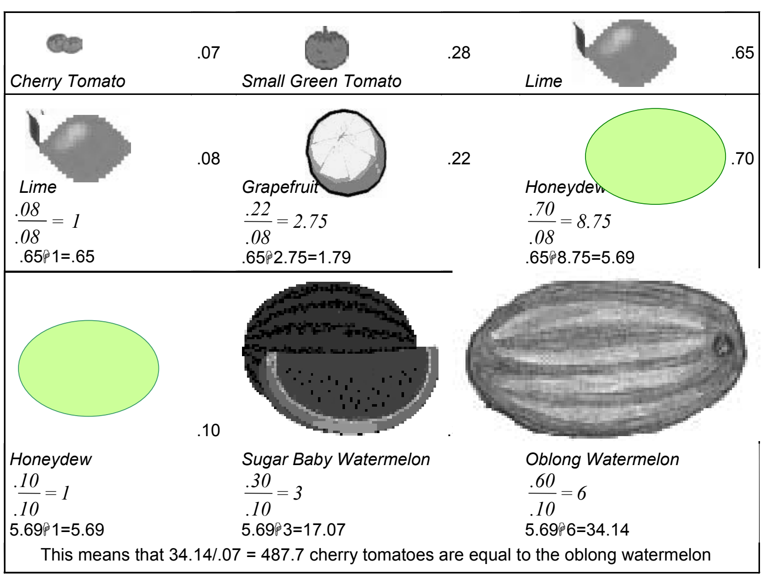
otas:

) El "Actual Results", se obtiene a partir del número de trabajadores actualmente afiliados a las diferentes mutuales (privadas), que administran

Extending the 1-9 Scale to 1- ∞

•The 1-9 AHP scale does not limit us if we know how to use clustering of similar objects in each group and use the largest element in a group as the smallest one in the next one. It serves as a pivot to connect the two.

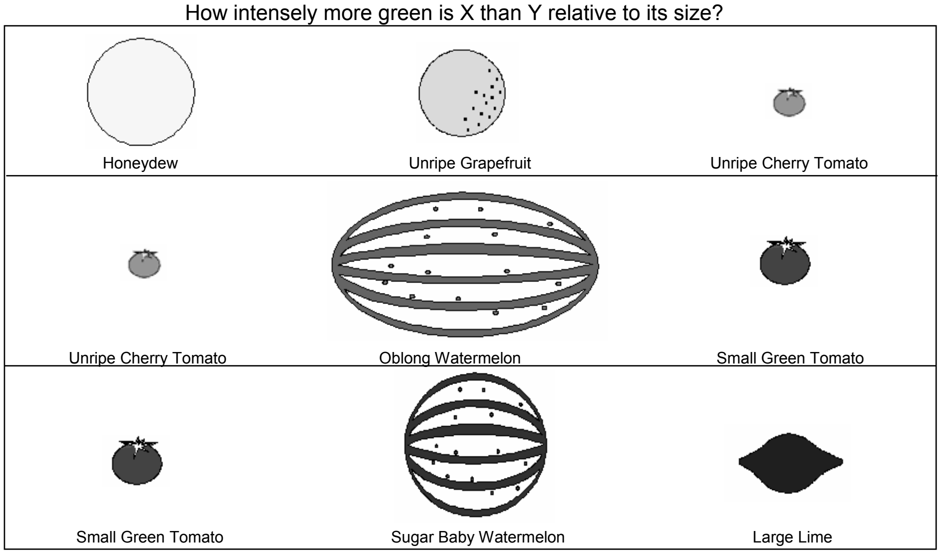
•We then compare the elements in each group on the 1-9 scale get the priorities, then divide by the weight of the pivot in that group and multiply by its weight from the previous group. We can then combine all the groups measurements as in the following example comparing a very small cherry tomato with a very large watermelon.



Clustering & Comparison

Color

How intensely more green is X than Y relative to its size?



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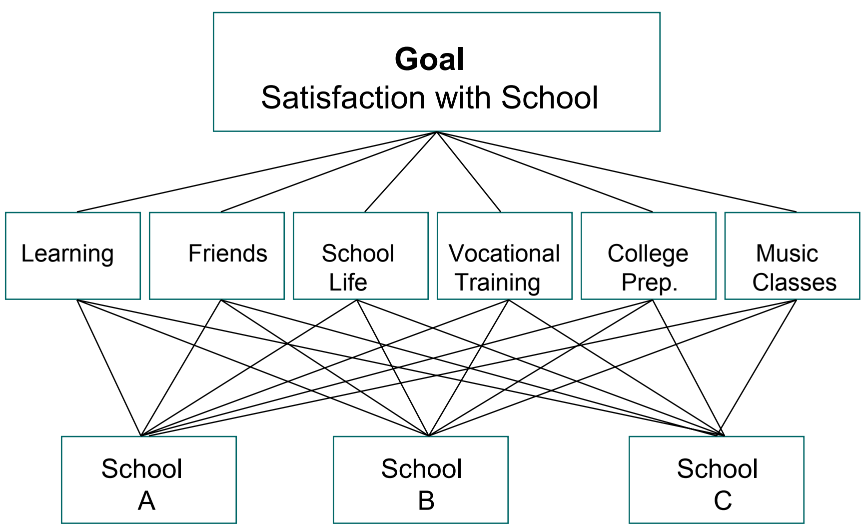


School Selection

	L	F	SL	VT	CP	MC	Weights
Learning	1	4	3	1	3	4	.32
Friends	1/4	1	7	3	1/5	1	.14
School Life	1/3	1/7	1	1/5	1/5	1/6	.03
Vocational Trng.	1	1/3	5	1	1	1/3	.13
College Prep.	1/3	5	5	1	1	3	.24
Music Classes	1/4	1	6	3	1/3	1	.14

Comparison of Schools with Respect to the Six Characteristics

Learning	A	B	C	Priorities
A	1	1/3	1/2	.16
B	3	1	3	.59
C	2	1/3	1	.25

Friends	A	B	C	Priorities
A	1	1	1	.33
B	1	1	1	.33
C	1	1	1	.33

School Life	A	B	C	Priorities
A	1	5	1	.45
B	1/5	1	1/5	.09
C	1	5	1	.46

Vocational Trng.	A	B	C	Priorities
A	1	9	7	.77
B	1/9	1	1/5	.05
C	1/7	5	1	.17

College Prep.	A	B	C	Priorities
A	1	1/2	1	.25
B	2	1	2	.50
C	1	1/2	1	.25

Music Classes	A	B	C	Priorities
A	1	6	4	.69
B	1/6	1	1/3	.09
C	1/4	3	1	.22

Composition and Synthesis

Impacts of School on Criteria

	.32 L	.14 F	.03 SL	.13 VT	.24 CP	.14 MC	**Composite Impact of Schools**
A	.16	.33	.45	.77	.25	.69	.37
B	.59	.33	.09	.05	.50	.09	.38
C	.25	.33	.46	.17	.25	.22	.25

The School Example Revisited Composition & Synthesis:

Impacts of Schools on Criteria

Distributive Mode
(Normalization: Dividing each entry by the total in its column)

	.32 L	.14 F	.03 SL	.13 VT	.24 CP	.14 MC	Composite Impact of Schools
A	.16	.33	.45	.77	.25	.69	.37
B	.59	.33	.09	.05	.50	.09	.38
C	.25	.33	.46	.17	.25	.22	.25

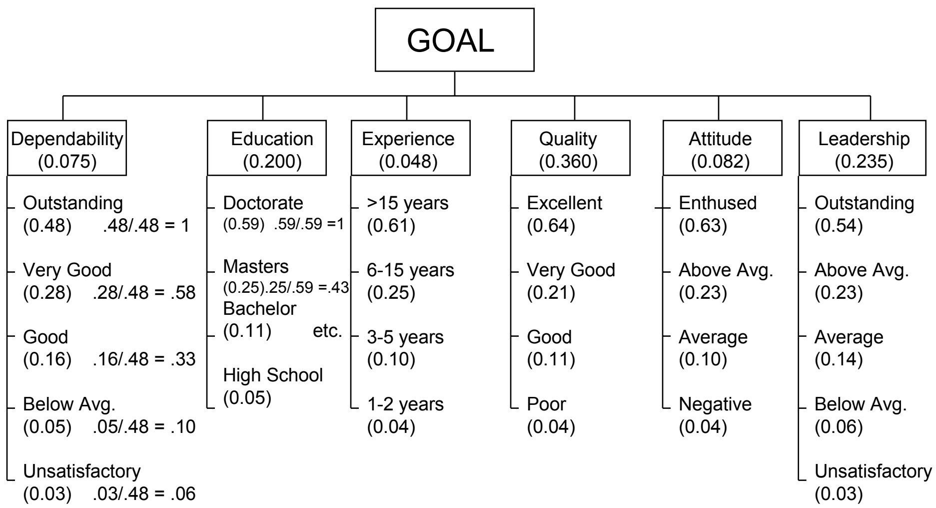
The Distributive mode is useful when the uniqueness of an alternative affects its rank. The number of copies of each alternative also affects the share each receives in allocating a resource. In planning, the scenarios considered must be comprehensive and hence their priorities depend on how many there are. This mode is essential for ranking criteria and sub-criteria, and when there is dependence.

Ideal Mode
(Dividing each entry by the maximum value in its column)

	.32 L	.14 F	.03 SL	.13 VT	.24 CP	.14 MC	Composite Impact of Schools	Normal-ized
A	.27	1	.98	1	.50	1	.65	.34
B	1	1	.20	.07	.50	.13	.73	.39
C	.42	1	1	.22	.50	.32	.50	.27

The Ideal mode is useful in choosing a best alternative regardless of how many other similar alternatives there are.

Evaluating Employees for Raises

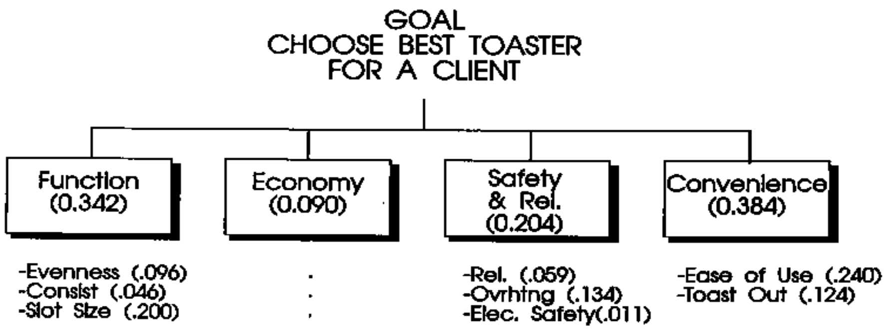


Final Step in Absolute Measurement

Rate each employee for dependability, education, experience, quality of work, attitude toward job, and leadership abilities.

	Dependability 0.0746	Education 0.2004	Experience 0.0482	Quality 0.3604	Attitude 0.0816	Leadership 0.2348	Total	Normalized
Esselman, T.	Outstand	Doctorate	>15 years	Excellent	Enthused	Outstand	1.000	0.153
Peters, T.	Outstand	Masters	>15 years	Excellent	Enthused	Abv. Avg.	0.752	0.115
Hayat, F.	Outstand	Masters	>15 years	V. Good	Enthused	Outstand	0.641	0.098
Becker, L.	Outstand	Bachelor	6-15 years	Excellent	Abv. Avg.	Average	0.580	0.089
Adams, V.	Good	Bachelor	1-2 years	Excellent	Enthused	Average	0.564	0.086
Kelly, S.	Good	Bachelor	3-5 years	Excellent	Average	Average	0.517	0.079
Joseph, M.	Blw Avg.	Hi School	3-5 years	Excellent	Average	Average	0.467	0.071
Tobias, K.	Outstand	Masters	3-5 years	V. Good	Enthused	Abv. Avg.	0.466	0.071
Washington, S.	V. Good	Masters	3-5 years	V. Good	Enthused	Abv. Avg.	0.435	0.066
O'Shea, K.	Outstand	Hi School	>15 years	V. Good	Enthused	Average	0.397	0.061
Williams, E.	Outstand	Masters	1-2 years	V. Good	Abv. Avg.	Average	0.368	0.056
Golden, B.	V. Good	Bachelor	.15 years	V. Good	Average	Abv. Avg.	0.354	0.054

The total score is the sum of the weighted scores of the ratings. The money for raises is allocated according to the normalized total score. In practice different jobs need different hierarchies.

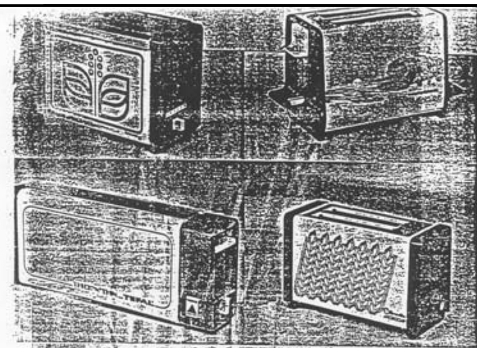


Toasters were assigned 0 or 1 under subcriteria; Econ=1 if Cost<17 Brit. pounds

6-04-96

TOASTER RATINGS

	EVEN-NESS .096	CON-SIST. .046	SLOT SIZE .200	PRICE .090	REL .059	OVR/HTS .134	ELEC SAFTY .011	EASE USE .240	TOAST OUT .124	TOT.
BREVILLE PUT 1	0	0	0	1	0	1	0	0	1	.344
BREVILLE PUT 2	0	0	0	1	1	1	0	1	0	.524
BREVILLE PUT 3	0	0	0	1	1	1	0	1	1	.644
CURRYS TYPE 20	0	0	0	1	1	1	0	0	1	.404
INDESIT 2TI	0	0	0	1	0	1	0	1	1	.584
MORPHY 44302	0	0	0	1	1	1	0	1	1	.644
NATIONAL (TAI.)	0	1	0	1	1	1	0	1	1	.690
PHILLIPS '41	1	1	0	1	0	1	0	0	0	.366
PHILLIPS '45	0	0	0	1	1	1	0	1	1	.644
RUSSELL HOBBS66	1	0	0	0	1	0	0	1	0	.396
SALTON T30	0	0	0	1	1	1	0	1	1	.644
SPINNEY	0	0	0	1	0	1	0	1	1	.584
SWAN '52	0	0	0	0	0	1	0	1	0	.374
SUNBEAM 83	0	0	0	1	0	1	0	1	0	.464
TEFAL	0	0	0	0	1	1	0	0	1	.314
4 SLICE TOASTERS										
MORPHY 44902	0	0	1	0	1	1	0	1	1	.620
RUSSELL HOBBS 75	0	0	1	0	1	0	0	1	0	.500
SWAN '54	0	0	1	0	1	0	0	1	0	.500
SUNBEAM 84	0	0	1	0	1	1	0	1	0	.634



The toasters pictured here did best all round in our tests and had no serious drawbacks. They are: National NT124 (top left); Morphy Richards Harvest 44302 (top right); Tefal Thick and Thin 8426 (bottom left); Breville PUT 3 Microchip (bottom right).

Similar models

Below, in bold, we list models which are very similar to ones tested for this report, with likely prices and, if relevant, how they differ from our tested toasters.

Kenwood 122 (£17) – Breville PUT 1.

Indesit 472 (£21) – 4-slice version of Indesit 2T1.

ADL 44330 (£12, Argos); **Euro** (£11, Comet); **Exclusive** (£14, Electricity Boards); **Morphy Richards 44300** (£15); **Pilot 44340** (£11, Rumbelows) – all similar to Morphy Richards 44302.

Euro (£17, Comet); **Morphy Richards 44900** (£20); **Pilot 44940** (£19, Rumbelows) – all similar to Morphy Richards 44902.

Russell Hobbs 5453; 5456; 5457; 5458; 5460 (£20) – all same as Russell Hobbs 5455 but with different patterns or finish.

Russell Hobbs 5477 (£24) – Russell Hobbs 5475.

Salton T30 series includes T31; T32; T33; T33b; T34; T35; T35m; T36 as well as the T30 – all different patterns and finishes.

Sona T34 – same as Salton T30.

Spinney PJ131 (£20) – 4-slice version of PJ132.

Swan 20451 – Swan 20452, but with different finish; same price.

Swan 20499 (£22) – Swan 20454, but with different finish.

Pop-up toasters

	target price £	speed of toasting 1st batch min	sec	4th batch min	sec	convenience
2-SLICE TOASTERS						
Breville PUT 1 Type 28 (W Germany) [1]	10	2	00	1	40	□
Breville PUT 2 (Hong Kong) [1]	14	2	00	1	40	□
Breville PUT 3 Microchip Type 969 (W Germany)	16	2	20	1	50	◩
Currys Type 25 (W Germany)	11	2	00	1	40	□
Indesit 2T1 (KS426) (Hong Kong)	12	2	40	1	50	◪
Morphy Richards Harvest 44302 (UK)	15	2	10	2	20	◨
National NT124 (Taiwan)	16	2	30	1	50	◩
Philips HD4541/A (Austria)	15	4	00	2	30	◪
Philips HD4545/A Coldwall (Austria)	15	3	00	2	00	◪
Russell Hobbs Microchip 5455 (UK)	20	2	10	1	50	◨
Salton T30/Sona (W Germany)	14	2	10	1	30	□
Spinney PJ132/25315 (Hong Kong) (Littlewoods Mail Order) [2]	14	1	50	1	20	◪
Swan 20452 (Hong Kong) [3]	17	2	10	1	40	□
Sunbeam Auto 083 (Hong Kong)	13	2	20	1	50	◪
Tefal Thick and Thin 8426 (France)	17	1	50	1	20	□
4-SLICE TOASTERS						
Morphy Richards Harvest 44902 (UK)	20	2	50	1	40	◨
Russell Hobbs Microchip 5475 (UK)	24	2	40	2	20	◨
Swan 20454 (Hong Kong) [3]	21	2	30	1	50	□
Sunbeam Auto 084 (Hong Kong)	19	2	40	2	10	◪

[1] [illegible] [2] [illegible] [3] [illegible]

Toasters picked out in the Table in yellow bands are those which appear in the Buying Guide.

KEY TO RATINGS ◨ ◩ □ ◪ ■ best ←——→ worst

TOASTERS

Nineteen tested. Overall, a poor performance for such simple machines. But four came out best all round

Toasters have a simple function: to make evenly browned toast quickly and reliably. But for such simple machines, many of them seem remarkably poor at their job. Pop-up mechanisms have a habit of going wrong and brownness of toast is often uneven and inconsistent. Outside surfaces get too hot for comfort and some slots are too small to take your favourite slice or bun (eg half a bap). For this report we tested 19 toasters. We wanted to find out:

- which toasters consistently made the most evenly toasted bread
- how long they took
- which toasters could take the largest and thickest slices
- which toasters were most reliable
- which were the easiest to clean and look after.

Evenness of toast
We assessed the toast from each toaster for evenness of browning including an assessment of how 'striped' the toast was. All the toasters gave variable results, but the Philips HD4541 and Russell Hobbs Microchip 5455 produced slightly more even toast than the others.

Consistency
All the toasters had a browning control which should allow you to get the toast the way you like it, whatever the bread. We found that with the control set first at minimum and then at maximum, all the toasters produced pale or burnt toast respectively. We then toasted four batches with the browning control set to give a good medium brown toast. The different batches were assessed for amount of browning. Two toasters – National NT124 and Philips HD4541 – gave more consistent results than the others.

Speed
We timed how long it took to toast a batch of two (or four) slices of bread with the control set to give medium brown toast. We did this three times and give in the Table the times taken for the first and fourth batches.

Slot size
To see what limits the slots imposed on each toaster, we tried thick-sliced white Sunblest bread and a standard Hovis tin. Our white bread was slightly oblong in shape so we tried fitting it in both upright and on its side. We also checked to see which toasters could handle a crumpet or half a bap.

Few of the toasters comfortably took all our food. The crumpets and half-baps had to be squeezed into most toasters and a few couldn't take them at all. The Tefal toaster was the most versatile – its slot was very wide but was self-adjusting to cope with thinner slices. The Philips HD4541 and the two Russell Hobbs coped slightly better with thick slices than the remainder of the toasters.

Reliability
We put two samples of each toaster through 4000 cycles of toasting, equivalent to about five years' use. Most toasters came through unscathed but both samples of Breville PUT 1, Indesit, Spinney and Sunbeam 083 failed during the test, as did one sample each of Philips HD4541 and Swan 20452.

Convenience
We assessed the ease with which the browning control could be set; how easy each toaster was to load; and whether you could manually eject the toast using only one hand (to avoid touching hot parts with the other hand). Also included in the rating in the Table was ease of cleaning, both the exterior of the toaster and the interior crumb tray.

All but the Breville PUT 1, Currys, Philips HD4541 and Tefal toasters had removable crumb trays. You had to shake these four toasters upside down to remove crumbs – a disadvantage.

Most toasters provided about one metre of mains flex. The Breville PUT 1 and PUT 3, Currys, both Philips and Tefal provided more – around 1.35 metres. The Salton's flex was the shortest – only 0.75m.

Getting overheated
Obviously a toaster gets very hot in parts when it is working. But the walls and controls should stay cool enough to be touchable without burning yourself.

All the toasters were cool enough to be safe. But the plastic end moulding (opposite the end with the controls) on the two Russell Hobbs toasters and the Swan 4-slice got uncomfortably, though not dangerously, hot. (Swan tell us they have modified the moulding to get over this problem.)

Taking toast out
None of the toasters gave any problems with normal slices of bread. But our crumpets and half-baps didn't pop up high enough to protrude above the top of the toasting slots in the Breville PUT 2, Philips HD4541, both Russell Hobbs, both Swans and both Sunbeams. Not only was this inconvenient, but you stood a chance of burning your fingers; and if you used a fork, for example, to prise out a crumpet, you could even damage the element.

Electrical safety
All the toasters passed our safety tests.

BUYING GUIDE

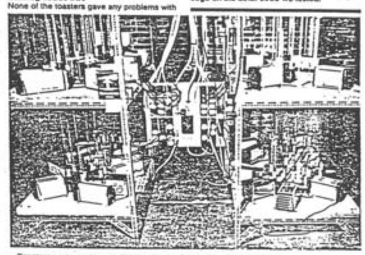
We didn't find the perfect toaster. Some browned toast more evenly than others; some toasted more consistently; some could take a wider variety of bread (including crumpets etc); others were more reliable.

We list below toasters which performed reasonably well all round with no serious drawbacks.

Breville PUT 3 Microchip (£16), **Morphy Richards Harvest 44302** (£15), **National NT124** (£16), **Tefal Thick and Thin 8426** (£17).

The **Philips HD4541/A** (£15) and **Russell Hobbs Microchip 5455** (£20) made marginally better toast than those recommended above, but had drawbacks which might upset some users.

If you want a four-slice toaster, the **Morphy Richards 44902** (£20) just had the edge on the other three we tested.



Toasters under test: our reliability rig simulates about five years' use

Similar models

Below, in bold, we list models which are very similar to ones tested for this report, with likely prices and, if relevant, how they differ from our tested toasters.

Kenwood 122 (£17) – Breville PUT 1.

Indesit 4T2 (£19) – 4-slice version of Indesit 2T1.

ADL 44330 (£12, Argos); **Euro** (£11, Comet); **Exclusive** (£14, Electricity Boards); **Morphy Richards 44300** (£15); **Pilot 44340** (£11, Rumbelows) – all similar to Morphy Richards 44302.

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Russell Hobbs 5477 (£24) – Russell Hobbs 5475.

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Sona T34 – same as Salton T30.

Spinney PJ131 (£20) – 4-slice version of PJ132.

Swan 20451 – Swan 20452, but with different finish; same price.

Swan 20499 (£22) – Swan 20454, but with different finish.

Pop-up toasters

	target price £	speed of toasting 1st batch min	1st batch sec	4th batch min	4th batch sec	convenience
2-SLICE TOASTERS						
Breville PUT 1 Type 20 (W Germany) [1]	10	2	00	1	40	□
Breville PUT 2 (Hong Kong) [1]	14	2	00	1	40	□
Breville PUT 3 Microchip Type 909 (W Germany)	16	2	20	1	50	◩
Currys Type 20 (W Germany)	11	2	00	1	40	□
Indesit 2T1 (KS428) (Hong Kong)	12	2	40	1	50	◪
Morphy Richards Harvest 44302 (UK)	15	2	10	2	20	▨
National NT124 (Taiwan)	16	2	30	1	50	◩
Philips HD4541/A (Austria)	15	4	00	2	30	◪
Philips HD4545/A Coldwall (Austria)	15	3	00	2	00	◪
Russell Hobbs Microchip 5455 (UK)	20	2	10	1	50	▨
Salton T30/Sona (W Germany)	14	2	10	1	30	□
Spinney PJ132/20315 (Hong Kong) (Littlewoods Mail Order) [2]	14	1	50	1	20	◪
Swan 20452 (Hong Kong) [3]	17	2	10	1	40	□
Sunbeam Auto 083 (Hong Kong)	13	2	20	1	50	◪
Tefal Thick and Thin 8426 (France)	17	1	50	1	20	□
4-SLICE TOASTERS						
Morphy Richards Harvest 44902 (UK)	20	2	60	1	40	▨
Russell Hobbs Microchip 5475 (UK)	24	2	40	2	20	▨
Swan 20454 (Hong Kong) [3]	21	2	30	1	50	□
Sunbeam Auto 084 (Hong Kong)	19	2	40	2	10	◪

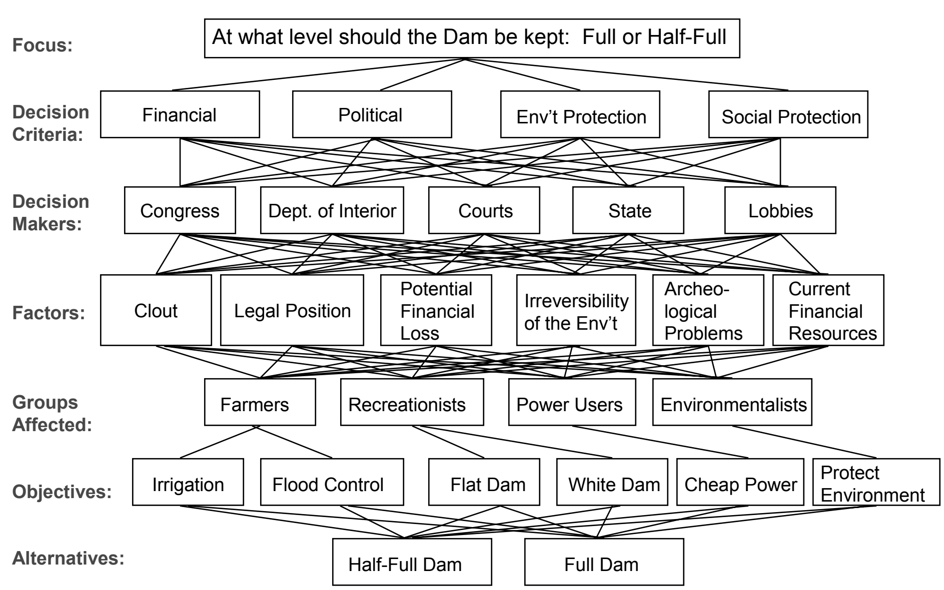
[1] [illegible] [2] [illegible] [3] [illegible]

Toasters picked out in the Table in yellow bands are those which appear in the Buying Guide.

KEY TO RATINGS ▨ ◩ □ ◪ ■ best ⟵⟶ worst

Which? May 1985 Toasters 223

A Complete Hierarchy to Level of Objectives



Should U.S. Sanction China? (Feb. 26, 1995)

BENEFITS

Protect rights and maintain high Incentive to make and sell products in China (0.696)	Rule of Law Bring China to responsible free-trading 0.206)	Help trade deficit with China (0.098)
Yes .80	Yes .60	Yes .50
No .20	No .40	No .50

Yes 0.729 **No 0.271**

COSTS

$ Billion Tariffs make Chinese products more expensive (0.094)	Retaliation (0.280)	Being locked out of big infrastructure buying: power stations, airports (0.626)
Yes .70	Yes .90	Yes .75
No .30	No .10	No .25

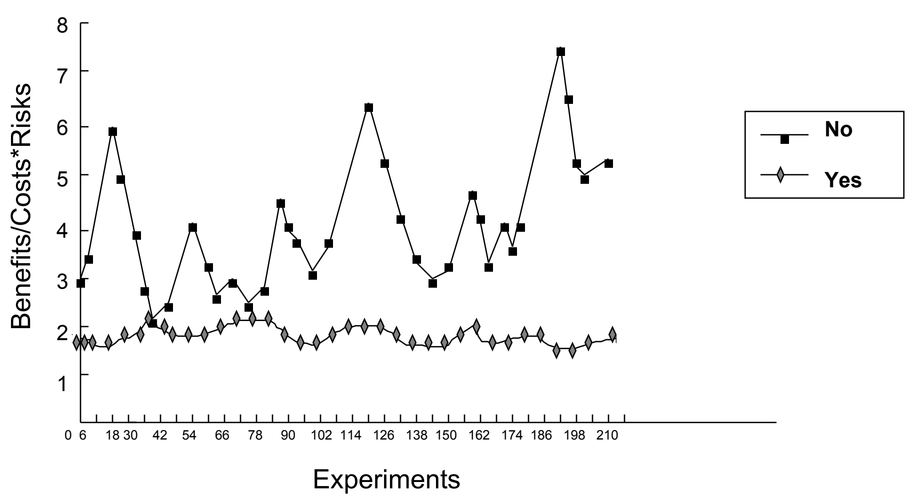
Yes 0.787 **No 0.213**

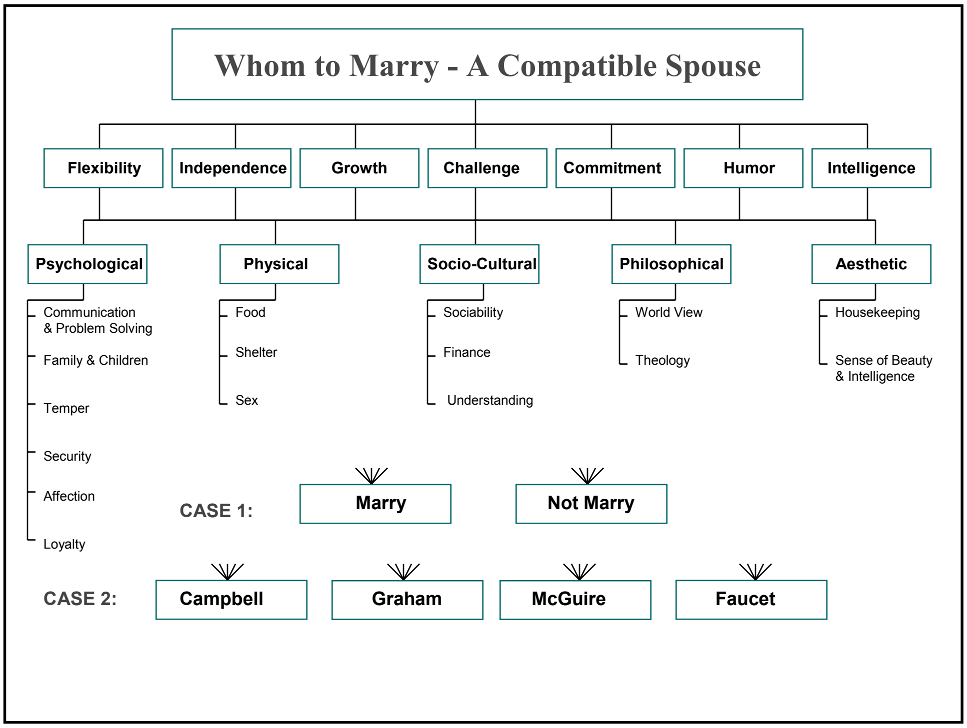
RISKS

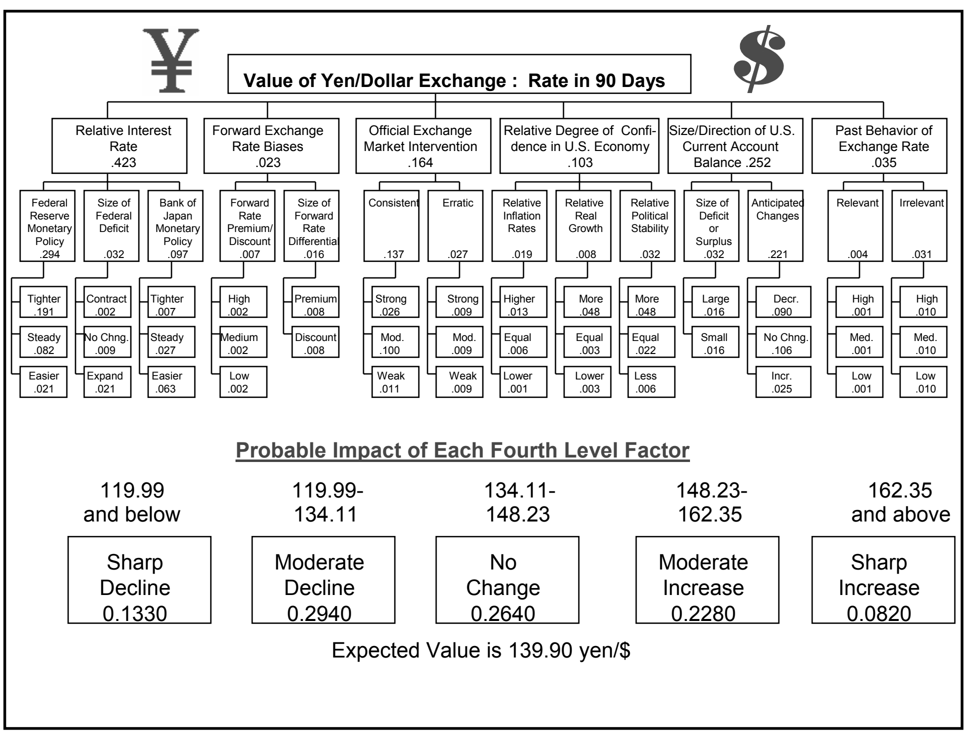
Long Term negative competition (0.683)	Effect on human rights and other issues (0.200)	Harder to justify China joining WTO (0.117)
Yes .70	Yes .30	Yes .50
No .30	No .70	No .50

Yes 0.597 **No 0.403**

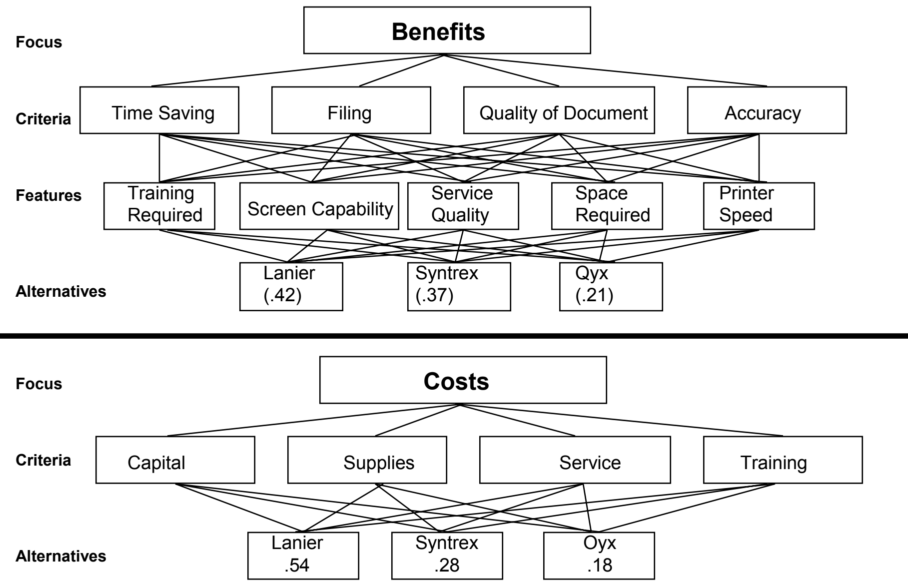
Result: $\frac{\text{Benefits}}{\text{Costs x Risks}}$; **YES** $\frac{.729}{.787 \times .597} = 1.55$ **NO** $\frac{.271}{.213 \times .403} = 3.16$




Whom to Marry - A Compatible Spouse
Flexibility
Independence
Growth
Challenge
Commitment
Humor
Intelligence
Psychological
Physical
Socio-Cultural
Philosophical
Aesthetic
Communication & Problem Solving
Family & Children
Temper
Security
Affection
Loyalty
Food
Shelter
Sex
Sociability
Finance
Understanding
World View
Theology
Housekeeping
Sense of Beauty & Intelligence
CASE 1:
Marry
Not Marry
CASE 2:
Campbell
Graham
McGuire
Faucet


Value of Yen/Dollar Exchange : Rate in 90 Days
Relative Interest Rate .423
Forward Exchange Rate Biases .023
Official Exchange Market Intervention .164
Relative Degree of Confidence in U.S. Economy .103
Size/Direction of U.S. Current Account Balance .252
Past Behavior of Exchange Rate .035
Federal Reserve Monetary Policy .294
Size of Federal Deficit .032
Bank of Japan Monetary Policy .097
Forward Rate Premium/ Discount .007
Size of Forward Rate Differential .016
Consistent .137
Erratic .027
Relative Inflation Rates .019
Relative Real Growth .008
Relative Political Stability .032
Size of Deficit or Surplus .032
Anticipated Changes .221
Relevant .004
Irrelevant .031
Tighter .191
Steady .082
Easier .021
Contract .002
No Chng. .009
Expand .021
Tighter .007
Steady .027
Easier .063
High .002
Medium .002
Low .002
Premium .008
Discount .008
Strong .026
Mod. .100
Weak .011
Strong .009
Mod. .009
Weak .009
Higher .013
Equal .006
Lower .001
More .048
Equal .003
Lower .003
More .048
Equal .022
Less .006
Large .016
Small .016
Decr. .090
No Chng. .106
Incr. .025
High .001
Med. .001
Low .001
High .010
Med. .010
Low .010
Probable Impact of Each Fourth Level Factor
119.99 and below
119.99- 134.11
134.11- 148.23
148.23- 162.35
162.35 and above
Sharp Decline 0.1330
Moderate Decline 0.2940
No Change 0.2640
Moderate Increase 0.2280
Sharp Increase 0.0820
Expected Value is 139.90 yen/$

Best Word Processing Equipment



Best Word Processing Equipment Cont.

Benefit/Cost Preference Ratios

Lanier	Syntrex	Qyx
$\frac{.42}{.54} = 0.78$	$\frac{.37}{.28} = 1.32$	$\frac{.21}{.18} = 1.17$

Best Alternative

Group Decision Making and the Geometric Mean

Suppose two people compare two apples and provide the judgments for the larger over the smaller, 4 and 3 respectively. So the judgments about the smaller relative to the larger are 1/4 and 1/3.

<u>Arithmetic mean</u>

$$4 + 3 = 7$$

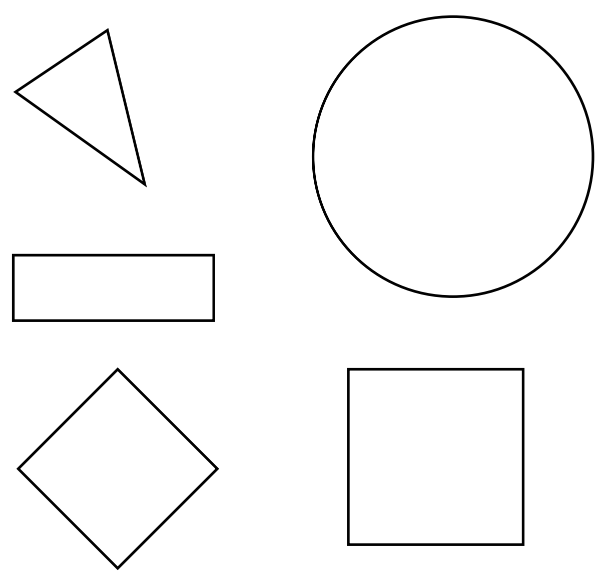
$$1/7 \neq 1/4 + 1/3 = 7/12$$

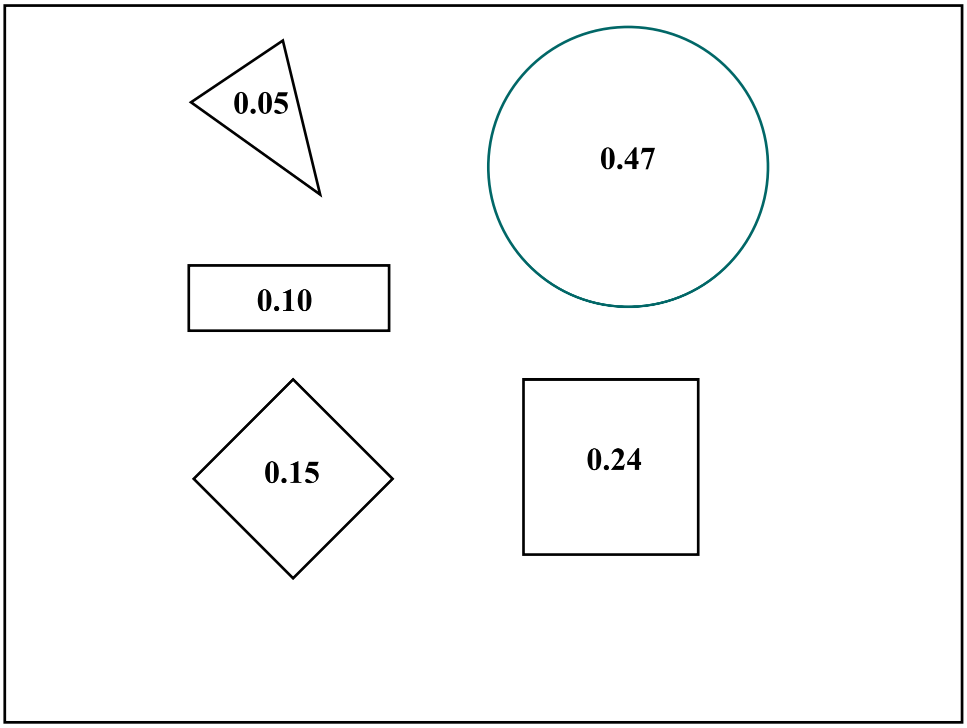
<u>Geometric mean</u>

$$\sqrt{4 \times 3} = 3.46$$

$$1/\sqrt{4 \times 3} = \sqrt{1/4 \times 1/3} = 1/\sqrt{4 \times 3} = 1/3.46$$

That the Geometric Mean is the unique way to combine group judgments is a theorem in mathematics.





ASSIGNING NUMBERS vs. PAIRED COMPARISONS

- A number assigned directly to an object is at best an ordinal and cannot be justified.
- When we compare two objects or ideas we use the smaller as a unit and estimate the larger as a multiple of that unit.

• If the objects are homogeneous and if we have knowledge and experience, paired comparisons actually derive measurements that are likely to be close and that indicate magnitude on a ratio scale.

PROBLEMS OF UTILITY THEORY

1. Utility theory is normative; it pre-scribes technically how “rational behavior” should be rather than looking at how people behave in making decisions.
2. Utility theory regards a criterion as important if it has alternatives well spread on it. Later it adopted AHP prioritization of criteria.

3. Alternatives are measured on an interval scale. Interval scale numbers can't be added or multiplied and are useless in resource allocation and dependence and feedback decisions.
4. Utility theory can only deal with a two-level structures if it is to use interval scales throughout.

5. Alternatives are rated one at a time on standards, and are never compared directly with each other.
6. It's implementation relies on the concept of lotteries (changed to value functions) which are difficult to apply to real life situations.
7. Until the AHP showed how to do it, utility theory could not cope precisely with intangible criteria.
8. Utility theory has paradoxes. (Allais showed people don't work

WHY IS AHP EASY TO USE?

• It does not take for granted the measurements on scales, but asks that scale values be interpreted according to the objectives of the problem.

• It relies on elaborate hierarchic structures to represent decision problems and is able to handle problems of risk, conflict, and prediction.

• It can be used to make direct resource allocation, benefit/cost analysis, resolve conflicts, design and optimize systems.

• It is an approach that describes how good decisions are made rather than prescribes how they should be made.

WHY THE AHP IS POWERFUL IN CORPORATE PLANNING

1. Breaks down criteria into manageable components.
2. Leads a group into making a specific decision for consensus or tradeoff.
3. Provides opportunity to examine disagreements and stimulate discussion and opinion.
4. Offers opportunity to change criteria, modify judgments.
5. Forces one to face the entire problem at once.
6. Offers an actual measurement system. It enables one to estimate relative magnitudes and derive ratio scale priorities accurately.

7. It organizes, prioritizes and synthesizes complexity within a rational framework.

8. Interprets experience in a relevant way without reliance on a black box technique like a utility function.

9. Makes it possible to deal with conflicts in perception and in judgment.

$$
\begin{array}{c}
 \\ A_1 \\ \vdots \\ A_n
\end{array}
\begin{array}{c}
\begin{array}{ccc} A_1 & \cdots & A_n \end{array} \\
\begin{bmatrix}
\frac{w_1}{w_1} & \cdots & \frac{w_1}{w_n} \\
\vdots & & \vdots \\
\frac{w_n}{w_1} & \cdots & \frac{w_n}{w_n}
\end{bmatrix}
\end{array}
\begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = n \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}
$$

$$A\, w = n\, w$$

A is consistent if its entries satisfy the condition

$$a_{jk} = a_{ik}/a_{ij}.$$

Theorem: *A positive n by n matrix has the ratio form $A = (w_i/w_j)$, $i,j = 1,\ldots,n$, if, and only if, it is consistent.*

Theorem: *The matrix of ratios $A = (w_i/w_j)$ is consistent if and only if n is its principal eigenvalue and $Aw = nw$. Further, $w > 0$ is unique to within a multiplicative constant.*

When A is inconsistent we write $a_{ij} = (w_i/w_j)\varepsilon_{ij}$, $E = (\varepsilon_{ij})$, $e^T = (1,\ldots,1)$

Theorem: *w is the principal eigenvector of a positive matrix A if and only if* $Ee = \lambda_{max}e$.

When the matrix *A* is inconsistent we have:

Theorem: $\lambda_{max} \geq n$

Proof: Using $a_{ji} = 1/a_{ij}$, and $Aw = \lambda_{max}w$, we have

$$\lambda_{max} - n = (1/n) \sum_{1 \leq i \leq j \leq n} [\delta^2_{ij} / (1 + \delta_{ij})] \geq 0$$

where $a_{ij} = (1+\delta_{ij})(w_i / w_j)$, $\delta_{ij} > -1$

$$\sum_{j=1}^{n} a_{ij}\, w_j = \lambda_{\max}\, w_i$$

$$a_{ji} = 1/\, a_{ij}$$

$$\sum_{i=1}^{n} w_i = 1$$

$$\int_a^b K(s,t)\, w(t)\, dt = \lambda_{\max}\, w(s)$$

$$\lambda \int_a^b K(s,t) w(t) dt = w(s)$$

$$\int_a^b w(s) ds = 1$$

$K(s,t)\ K(t,s) = 1$

$K(s,t)\ K(t,u) = K(s,u)$,
for all s, t, and u

A consistent kernel satisfies

$$K(s,t)=k(s)/k(t)$$

From which the response eigenfunction w(s) is given by

$$w(s) = \frac{k(s)}{\int_S k(s)ds}$$

Thus $w(s)$ = ✂ $k(s)$

Generalizing on the discrete approach we assume that $K(s,t)$ is homogeneous of order 1. Thus, we have:

$$K(as, at) = a\,K(s,t) = k(as)/k(at) = a\,k(s)/k(t)$$

It turns out that the response eigenfunction w(s) satisfies the following functional equation

$$w(as) = bw(s)$$

where $b = \alpha a$.

The solution to this functional equation is also the solution of Fredholm's equation and is given by the general damped periodic response eigenfunction $w(s)$:

$$w(s) = Ce^{\log b \frac{\log s}{\log a}} P\left(\frac{\log s}{\log a}\right)$$

where P is periodic of period 1 and P(0)=1.

The well-known Weber Fechner logarithmic law of response to stimuli can be obtained as a first order approximation to our eigenfunction:

$$v(u)=C_1\, e^{-\beta u}\, P(u) \approx C_2 \log s + C_3$$

where $P(u)$ is periodic of period 1,

$u=\log s/\log a$ and $\log ab$☜$-\beta$, $\beta>0$.

The integer valued scale can be derived from the Weber-Fechner Law as follows

$$M = a \log s + b, \quad a \neq 0$$

$$s_1 = s_0 + \Delta s_0 = s_0 + \frac{\Delta s_0}{s_0} s_0 = s_0(1+r)$$

$$s_2 = s_1 + \Delta s_1 = s_1(1+r) = s_0(1+r)^2 \equiv s_0 \alpha^2$$

$$s_n = s_{n-1}\alpha = s_0 \alpha^n \quad (n = 0, 1, 2, \ldots)$$

$$n = \frac{(\log s_n - \log s_0)}{\log \alpha}$$

We take the ratios M_i / M_1 , i=1,…,n of the responses:

$$M_1 = a \log \alpha,\ M_2 = 2a \log \alpha, \ldots ,$$
$$M_n = na \log \alpha.$$

thus obtaining the *integer* values of the Fundamental scale of the AHP: 1, 2, …,n.

The next step is to provide a framework to represent synthesis of derived scales in the case of feedback.

98

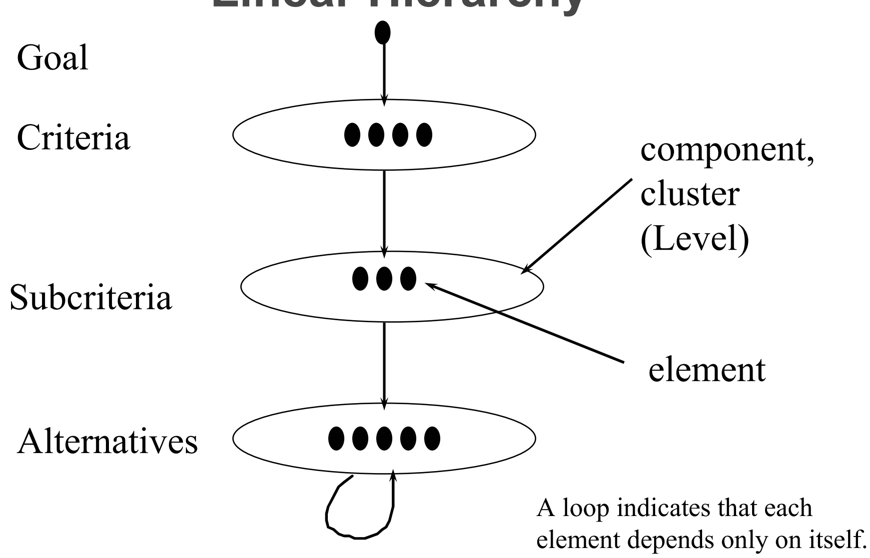
The Analytic Network Process (ANP) for Decision Making and Forecasting with Dependence and Feedback

- With feedback the alternatives depend on the criteria as in a hierarchy but may also depend on each other.
- The criteria themselves can depend on the alternatives and on each other as well.

- Feedback improves the priorities derived from judgments and makes prediction much more accurate.

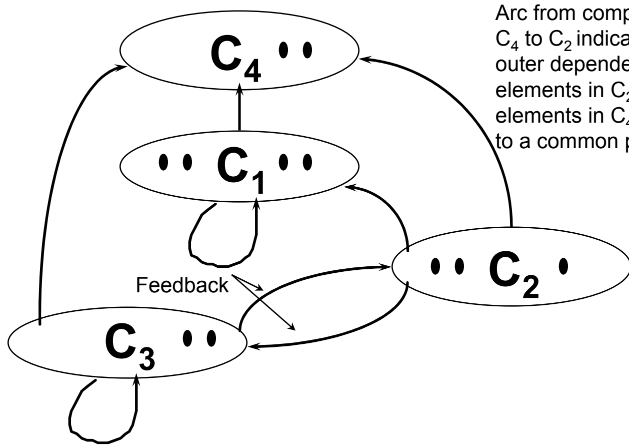
99

Linear Hierarchy



100

Feedback Network with components having Inner and Outer Dependence among Their Elements

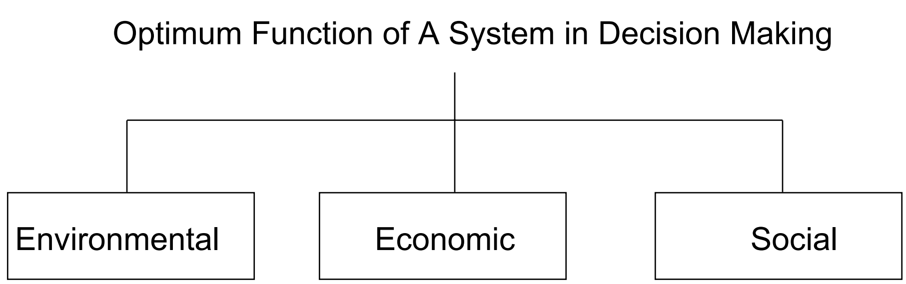


Arc from component C_4 to C_2 indicates the outer dependence of the elements in C_2 on the elements in C_4 with respect to a common property.

Loop in a component indicates inner dependence of the elements in that component with respect to a common property.

101

Example of Control Hierarchy



Influence is too general a concept and must be specified in terms of particular criteria. It is analyzed according to each criterion and then synthesized by weighting with these priorities of the "control" criteria belonging to a hierarchy or to a system.

102

Networks and the Supermatrix

$$
W = \begin{array}{cc}
 & \begin{array}{cccc} C_1 & C_2 & \cdots & C_N \\ e_{11}e_{12}\cdots e_{1n_1} & e_{21}e_{22}\cdots e_{2n_2} & & e_{N1}e_{N2}\cdots e_{Nn_N} \end{array} \\
\begin{array}{cc} C_1 & \begin{array}{c} e_{11} \\ e_{12} \\ \vdots \\ e_{1n_1} \end{array} \\ C_2 & \begin{array}{c} e_{21} \\ e_{22} \\ \vdots \\ e_{2n_2} \end{array} \\ \vdots & \\ C_N & \begin{array}{c} e_{N1} \\ e_{N2} \\ \vdots \\ e_{NuN} \end{array} \end{array} & \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1N} \\ W_{21} & W_{22} & \cdots & W_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ W_{N1} & W_{N2} & \cdots & W_{NN} \end{bmatrix}
\end{array}
$$

103

where

$$
W_{ij} = \begin{bmatrix} W_{i1}^{(j_1)} & W_{i1}^{(j_2)} & \cdots & W_{i1}^{(jn_j)} \\ W_{i2}^{(j_1)} & W_{i2}^{(j_2)} & \cdots & W_{i2}^{(jn_j)} \\ \vdots & \vdots & \cdots & \vdots \\ W_{in_i}^{(j_1)} & W_{in_i}^{(j_2)} & & W_{in_i}^{(jn_j)} \end{bmatrix}
$$

104

Predicted Turnaround Date of U.S. Economy from April 2001

2001 Prediction made April 7, 2001				
	Months	Midpoint	Priorities	Midpt x Priorities
Zero	0	0		
Three Months	3	1.5	0.20344	0.30516
Six Months	6	4.5	0.17022	0.76599
Twelve Months	12	9	0.21798	1.96182
Twenty Four Months	24	18	0.40846	7.35228
			SUM	**10.38525**

Turnaround of present slump in U.S. economy is predicted in about 10 months from April 2001 which would be around Feb. 2002

Supermatrix of a Hierarchy

$$
W = \begin{bmatrix}
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
W_{21} & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & W_{32} & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & W_{n-1,\, n-2} & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & W_{n,\, n-1} & I
\end{bmatrix}
$$

106

$$W^k = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ W_{n,n-1} W_{n-1,n-2} \ldots W_{32} W_{21} & W_{n,n-1} W_{n-1,n-2} \ldots W_{32} & \cdots & W_{n,n-1} W_{n-1,n-2} & W_{n,n-1} & I \end{bmatrix}$$

for k ≥ n

Hierarchic Synthesis

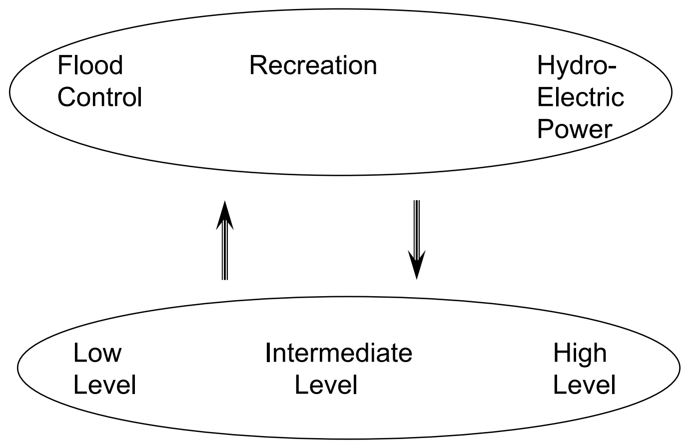
107

The Management of a Water Reservoir

Here we are faced with the decision to choose one of the possibilities of maintaining the water level in a dam at: Low (L), Medium (M) or High (H) depending on the relative importance of Flood Control (F), Recreation (R) and the generation of Hydroelectric Power (E) respectively for the three levels. The first set of three matrices gives the prioritization of the alternatives with respect to the criteria and the second set, those of the criteria in terms of the alternatives.

108

A Feedback System with Two Components



109

1) Which level is best for flood control?

Flood Control

	Low	Med	High	Eigenvector
Low	1	5	7	.722
Medium	1/5	1	4	.205
High	1/7	1/4	1	.073

Consistency Ratio = .107

2) Which level is best for recreation?

Recreation

	Low	Med	High	Eigenvector
Low	1	1/7	1/5	.072
Medium	7	1	3	.649
High	5	1/3	1	.279

Consistency Ratio = .056

3) Which level is best for power generation?

Power Generation

	Low	Med	High	Eigenvector
Low	1	1/5	1/9	.058
Medium	5	1	1/5	.207
High	9	5	1	.735

Consistency Ratio = .101

110

Low Level Dam

	F	R	E	Eigenvector
Flood Control	1	3	5	.637
Recreation	1/3	1	3	.258
Hydro-Electric Power	1/5	1/3	1	.105

Consistency Ratio = .033

1) At Low Level, which attribute is satisfied best?

2) At Intermediate Level, which attribute is satisfied best?

Intermediate Level Dam

	F	R	E	Eigenvector
Flood Control	1	1/3	1	.200
Recreation	3	1	3	.600
Hydro-Electric Power	1	1/3	1	.200

Consistency Ratio = .000

High Level Dam

	F	R	E	Eigenvector
Flood Control	1	1/5	1/9	.060
Recreation	5	1	1/4	.231
Hydro-Electric Power	9	4	1	.709

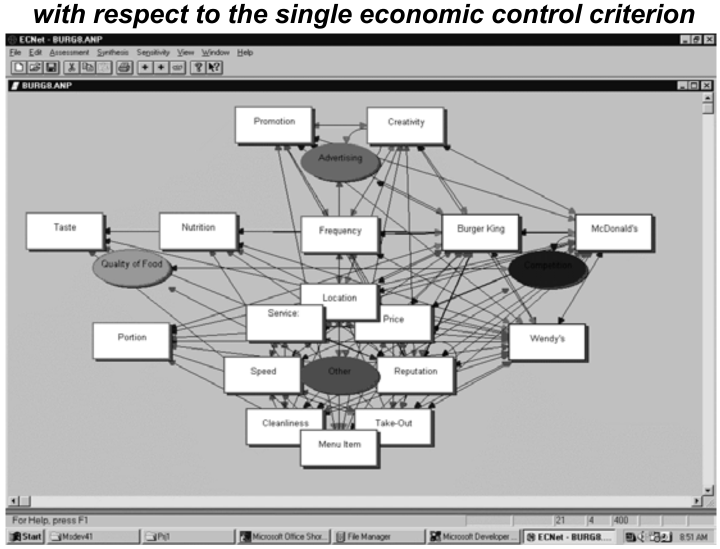
Consistency Ratio = .061

3) At High Level, which attribute is satisfied best?

111

Hamburger Model

Estimating Market Share of Wendy's, Burger King and McDonald's with respect to the single economic control criterion



112

Hamburger Model Supermatrix

		Other								Quality			Advertising			Competition		
	Local:	Menu	Cleanliness	Speed	Service	Location	Price	Reputation	Take Out	Portion	Taste	Nutrition	Frequency	Promotion	Creativity	Wendy's	Burger King	McDonald's
Other	Menu Item	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1930	0.0000	0.0000	0.0000	0.0000	0.3110	0.1670	0.1350	0.1570	0.0510	0.1590
	Cleanliness	0.6370	0.0000	0.0000	0.5190	0.0000	0.0000	0.2390	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2760	0.1100	0.3330
	Speed	0.1940	0.7500	0.0000	0.2850	0.0000	0.0000	0.0830	0.2900	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0640	0.1400	0.0480
	Service	0.0000	0.0780	0.1880	0.0000	0.0000	0.0000	0.0450	0.0550	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0650	0.1430	0.0240
	Location	0.0530	0.1710	0.0000	0.0980	0.0000	0.5000	0.2640	0.6550	0.0000	0.0000	0.0000	0.1960	0.0000	0.7100	0.1420	0.2240	0.1070
	Price	0.1170	0.0000	0.0000	0.0000	0.0000	0.0000	0.0620	0.0000	0.8570	0.0000	0.0000	0.0000	0.8330	0.0000	0.0300	0.2390	0.0330
	Reputation	0.0000	0.0000	0.0810	0.0980	0.0000	0.0000	0.0570	0.0000	0.0000	0.0000	0.0000	0.4930	0.0000	0.1550	0.2070	0.0420	0.2230
	Take-Out	0.0000	0.0000	0.7310	0.0000	0.0000	0.5000	0.0570	0.0000	0.1430	0.0000	0.0000	0.0000	0.0000	0.0000	0.0590	0.0510	0.0740
Q	Portion	0.2290	0.0000	0.0000	0.0000	0.0000	0.8330	0.2800	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0940	0.6490	0.5280
	Taste	0.6960	0.0000	0.0000	0.0000	0.0000	0.0000	0.6270	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2800	0.0720	0.1400
	Nutrition	0.0750	0.0000	0.0000	0.0000	0.0000	0.1670	0.0940	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6270	0.2790	0.3320
Ad	Frequency	0.7500	0.0000	0.0000	0.0000	0.0000	0.1670	0.5500	0.0000	0.0000	0.0000	0.0000	0.0000	0.6670	0.8750	0.6490	0.7090	0.6610
	Promotion	0.1710	0.0000	0.0000	0.0000	0.0000	0.8330	0.3680	0.0000	0.0000	0.0000	0.0000	0.5000	0.0000	0.1250	0.0720	0.1130	0.1310
	Creativity	0.0780	0.0000	0.0000	0.0000	0.0000	0.0000	0.0820	0.0000	0.0000	0.0000	0.0000	0.5000	0.3330	0.0000	0.2790	0.1790	0.2080
Comp	Wendy's	0.3110	0.5000	0.0990	0.5280	0.0950	0.0950	0.1010	0.1960	0.2760	0.6050	0.5940	0.0880	0.0880	0.1170	0.0000	0.1670	0.2000
	Burger King	0.1960	0.2500	0.3640	0.1400	0.2500	0.2500	0.2260	0.3110	0.1280	0.1050	0.1570	0.1950	0.1950	0.2680	0.2500	0.0000	0.8000
	McDonald's	0.4930	0.2500	0.5370	0.3330	0.6550	0.6550	0.6740	0.4940	0.5950	0.2910	0.2490	0.7170	0.7170	0.6140	0.7500	0.8330	0.0000

Cluster Priorities Matrix

Cluster:	Quality	Advertising	Competition	Other
Quality	0.000	0.000	0.066	0.066
Advertising	0.000	0.622	0.533	0.607
Competition	0.500	0.247	0.215	0.129
Other	0.500	0.131	0.187	0.198

113

Weighted Supermatrix

Weighted:	Menu	Cleanliness	Speed	Service	Location	Price	Reputation	Take Out	Portion	Taste	Nutrition	Frequency	Promotion	Creativity	Wendy's	Burger King	McDonald's
Menu Item	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0382	0.0000	0.0000	0.0000	0.0000	0.0407	0.0219	0.0177	0.0293	0.0095	0.0297
Cleanliness	0.1262	0.0000	0.0000	0.3141	0.0000	0.0000	0.0473	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0516	0.0205	0.0622
Speed	0.0384	0.4544	0.0000	0.1725	0.0000	0.0000	0.0164	0.1755	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0120	0.0261	0.0090
Service	0.0000	0.0473	0.1138	0.0000	0.0000	0.0000	0.0089	0.0333	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0121	0.0267	0.0045
Location	0.0105	0.1036	0.0000	0.0593	0.0000	0.0990	0.0523	0.3964	0.0000	0.0000	0.0000	0.0257	0.0000	0.0930	0.0265	0.0418	0.0200
Price	0.0232	0.0000	0.0000	0.0000	0.0000	0.0000	0.0123	0.0000	0.4287	0.0000	0.0000	0.0000	0.1091	0.0000	0.0056	0.0446	0.0062
Reputation	0.0000	0.0000	0.0490	0.0593	0.0000	0.0000	0.0113	0.0000	0.0000	0.0000	0.0000	0.0646	0.0000	0.0203	0.0387	0.0078	0.0417
Take-Out	0.0000	0.0000	0.4426	0.0000	0.0000	0.0990	0.0113	0.0000	0.0715	0.0000	0.0000	0.0000	0.0000	0.0000	0.0110	0.0095	0.0138
Portion	0.0151	0.0000	0.0000	0.0000	0.0000	0.0550	0.0185	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0062	0.0428	0.0348
Taste	0.0460	0.0000	0.0000	0.0000	0.0000	0.0000	0.0414	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0185	0.0047	0.0092
Nutrition	0.0050	0.0000	0.0000	0.0000	0.0000	0.0110	0.0062	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0413	0.0184	0.0219
Frequency	0.4554	0.0000	0.0000	0.0000	0.0000	0.1014	0.3338	0.0000	0.0000	0.0000	0.0000	0.0000	0.4149	0.5444	0.3455	0.3773	0.3519
Promotion	0.1038	0.0000	0.0000	0.0000	0.0000	0.5056	0.2233	0.0000	0.0000	0.0000	0.0000	0.3110	0.0000	0.0778	0.0383	0.0601	0.0697
Creativity	0.0474	0.0000	0.0000	0.0000	0.0000	0.0000	0.0498	0.0000	0.0000	0.0000	0.0000	0.3110	0.2071	0.0000	0.1485	0.0953	0.1107
Wendy's	0.0401	0.1974	0.0391	0.2082	0.0950	0.0123	0.0130	0.0773	0.1381	0.6044	0.5940	0.0217	0.0217	0.0289	0.0000	0.0359	0.0429
Burger King	0.0253	0.0987	0.1436	0.0552	0.2500	0.0323	0.0291	0.1226	0.0640	0.1049	0.1570	0.0482	0.0482	0.0662	0.0537	0.0000	0.1718
McDonald 's	0.0636	0.0987	0.2118	0.1313	0.6550	0.0845	0.0869	0.1948	0.2976	0.2907	0.2490	0.1771	0.1771	0.1517	0.1611	0.1788	0.0000

Limiting Supermatrix

Synthesized: Global	Menu	Cleanliness	Speed	Service	Location	Price	Reputation	Take Out	Portion	Taste	Nutrition	Frequency	Promotion	Creativity	Wendy's	Burger King	McDonald's
Menu Item	0.0234	0.0234	0.0234	0.0234	0.0234	0.0234	0.0234	0.0234	0.0234	0.0234	0.0234	0.0234	0.0234	0.0234	0.0234	0.0234	0.0234
Cleanliness	0.0203	0.0203	0.0203	0.0203	0.0203	0.0203	0.0203	0.0203	0.0203	0.0203	0.0203	0.0203	0.0203	0.0203	0.0203	0.0203	0.0203
Speed	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185
Service	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072
Location	0.0397	0.0397	0.0397	0.0397	0.0397	0.0397	0.0397	0.0397	0.0397	0.0397	0.0397	0.0397	0.0397	0.0397	0.0397	0.0397	0.0397
Price	0.0244	0.0244	0.0244	0.0244	0.0244	0.0244	0.0244	0.0244	0.0244	0.0244	0.0244	0.0244	0.0244	0.0244	0.0244	0.0244	0.0244
Reputation	0.0296	0.0296	0.0296	0.0296	0.0296	0.0296	0.0296	0.0296	0.0296	0.0296	0.0296	0.0296	0.0296	0.0296	0.0296	0.0296	0.0296
Take-Out	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152
Portion	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114
Taste	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049
Nutrition	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073
Frequency	0.2518	0.2518	0.2518	0.2518	0.2518	0.2518	0.2518	0.2518	0.2518	0.2518	0.2518	0.2518	0.2518	0.2518	0.2518	0.2518	0.2518
Promotion	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279
Creativity	0.1388	0.1388	0.1388	0.1388	0.1388	0.1388	0.1388	0.1388	0.1388	0.1388	0.1388	0.1388	0.1388	0.1388	0.1388	0.1388	0.1388
Wendy's	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435
Burger King	0.0784	0.0784	0.0784	0.0784	0.0784	0.0784	0.0784	0.0784	0.0784	0.0784	0.0784	0.0784	0.0784	0.0784	0.0784	0.0784	0.0784
McDonald's	0.1579	0.1579	0.1579	0.1579	0.1579	0.1579	0.1579	0.1579	0.1579	0.1579	0.1579	0.1579	0.1579	0.1579	0.1579	0.1579	0.1579

Relative local weights: Wendy's= 0.156, Burger King =0.281, and McDonald's=0.566

114

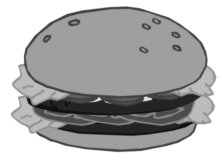
Hamburger Model

Synthesized Local:

Other	Menu Item	0.132
	Cleanliness	0.115
	Speed	0.104
	Service	0.040
	Location	0.224
	Price	0.138
	Reputation	0.167
	Take-Out	0.086
Quality	Portion	0.494
	Taste	0.214
	Nutrition	0.316

Synthesized Local Cont'd:

Advertising	Frequency	0.485
	Promotion	0.246
	Creativity	0.267
Competition	Wendy's	0.156
	Burger King	0.281
	McDonald's	0.566



	Simple Hierarchy (Three Level)	Complex Hierarchy (Several Levels)	Feedback Network	Actual Market Share
Wendy's	0.3055	0.1884	0.156	**0.1320**
Burger King	0.2305	0.2689	0.281	**0.2857**
McDonald's	0.4640	0.5427	0.566	**0.5823**

115

The Brain Hypermatrix
Order, Proportionality and Ratio Scales

- All order, whether in the physical world or in human thinking, involves proportionality among the parts, to establish harmony and synchrony among them in order to produce the whole.
- Proportionality means that there is a ratio relation among the parts. Thus, to study order or to create order, we must use ratio scales to capture and synthesize the relations inherent in that order. The question is how?
- We note that our perceptions of reality are miniaturized in our brains. We control the outside environment, which is much larger than the images we have of it, in a very precise way. This needs proportionality between what our brains perceive and how we interact with the outside world.

The Brain Hypermatrix and its Complex Valued Entries

The firings of a neuron are electrical signals. They have both a magnitude and a direction (a modulus and an argument) and are representable in the complex domain. We cannot do them justice by representing them with a real variable. Thus the mathematics of the brain must involve complex variables. The synthesis of signals requires proportionality among them. Such proportionality can be represented by a functional equation with a complex argument. Its solution represents the firings of a neuron and is what we want.

The Brain Hypermatrix and its Complex Valued Eigenfunction Entries

Generalizing on the real variable case involving Fredholm's equation of the second kind we begin with the basic proportionality functional equation:

$$w(az) = b\, w(z)$$

whose general solution with *a*, *b* and *z* complex is given by:

$$w(z) = C\, b^{(\log z / \log a)}\, P(\log z / \log a)$$

where P is an arbitrary multi-valued periodic function of period 1.

whose Fourier transform is given by:

$$= (1/2\pi)\log a \sum_{-\infty}^{\infty} a'_n \left[\frac{(2\pi n+\theta(b)-x)}{(\log a|b|+(2\pi n+\theta(b)-x)} \; i \right]$$

$$\delta(2\pi n+\theta(b)-x)$$

where $\delta(2\pi n+\theta(b)-x)$ is the Dirac delta function. In the real situation, the Fourier series is finite as the number of synapses and spines on a dendrite are finite.

There are three cases to consider in the solution of the functional equation *w(az)=bw(z)*.

1) That of real solutions;
2) That of complex solutions;
3) That of complex analytic solutions.

Here is a sketch of how the complex solution is derived. We choose the values of w arbitrarily in the ring between circles around 0 of radii 1 (incl.) and $|a|$ (excl.). We designate it by $W(z)$. Thus $w(z)=W(z)$ for $1 \leq |z| < |a|$. By the equation itself, $w(z) = w(z/a)\, b = W(z/a)\, b$ for

$|a| \leq |z| < |a|^2$, $w(z) = w(z/a)\, b = w(z/a^2)\, b^2 = W(z/a^2)\, b^2$

for $|a|^2 \leq |z| < |a|^3$, and so on (also $w(z) = w(az)/b = W(az)\, b^{-1}$ for $1/|a| \leq |z| < 1$ etc.). Thus the general complex solution of $w(az)=bw(z)$ is given by $w(z) = W(z/a^n)\, b^n$ for $|a|^n \leq |z| < |a|^{n+1}$ where $W(z)$ is arbitrary for $1 \leq |z| < |a|$. From, $|a|^n \leq |z|$ we have, $n = [\, \log |z| / \log |a|\,]$ where

$[\, x\,]$ is the integer closest to x from below. Here logarithms of real values are taken, so there are no multiple values to be concerned about. But then the solution becomes

$$w(z) = W\left(z/ a^{[\, \log |z| / \log |a|\,]}\right) b^{[\, \log |z| / \log |a|\,]},$$

with W arbitrary on the ring $1 \leq |z| < |a|$

Weierstrass' trigonometric approximation theorem:
Any complex-valued continuous function f(x) with period 2π can be approximated uniformly by a sequence of trigonometric polynomials of the form $\sum_n c_n e^{inx}$.

A function is called a periodic testing function if it is periodic and infinitely smooth. The space of all periodic testing functions with a fixed common period T is a linear space.
A distribution f is said to be periodic if there exists a positive number T such that $f(t) = f(t\text{-}T)$ for all T. This means that for every testing function ϕ $(f(t), \phi(t)) = (f(t-T), \phi(t))$.

Sobolev considered a Banach space of functions that are both Lebesgues integrable of class $p \geq 1$ and differentiable up to a certain order l and under certain conditions on p and l, also continuous.

Werner (1970) has shown that
(1) Every f(x) ε C[a,b] has a best [T-norm] approximation in E_n.
(2) If the best approximation to f(x) ε C[a,b] in E_n also belongs to E_n^0, then it is the unique best approximation.

A set of functions of the form $\sum_{k=1}^{n} c_k \; f_k(x)$, where c_k, k=1,…,n, are arbitrary reals and n=1,2,…, is dense in C[a,b], if the set of functions {f (x)} is closed in C[a,b], i.e., all its limit points belong to C[a,b].
Muntz proved that the set $\{s^{\alpha_k}\}$, $\alpha_k \geq 0$, k=1,2,…} is closed in C[a,b] if and only if $\sum (1/\alpha_k)$ diverges. Let t=-logs, it follows that the set $e^{-\beta_k t}$, $\beta_k \geq 0$, k=1,2,…} is closed in C[0,∞] if and only if $\sum (1/\beta_k)$ diverges. It can be shown that the set of products $\{ s^{\alpha_k} e^{-\beta_k t} \}$ is also closed in C[0,∞] with the same two conditions. Thus finite linear combinations of these functions are dense in C[0,∞].

The justification for the use of the gamma-like response functions $\{ s^{\alpha_k} e^{-\beta_k t} \}$ is partly theoretical and partly empirical. With the basic assumption that the decay of depolarization between impinging subthreshold impulses is negligible, the distribution of neuronal firing intervals in spontaneous activity has been approximated by the gamma distribution.
If the decay is not negligible as we assume in our work, then one can decompose the approximation into sums of exponentials as follows:

$$E_n^0 = \{f(x) \mid f(x)=\sum_{j=1}^{n} c_j e^{\lambda_j x} , c_j , \lambda_j \, \varepsilon \, R\}$$

However E_n^0 is not closed under the Tchebycheff or T-norm

$$||f(x)|| = \max_x | f(x) |$$

and hence a best approximation need not exist.

Several Ratio Scales and Related Functional Equations

One can multiply and divide but not add or subtract numbers from different ratio scales. We must synthesize different ratio scales that have the form of the eigenfunction solution

$$w_k(z_k) = (b_k)^{[\log|z_k|/\log|a_k|]} P_k(\log|z_k|/\log|a_k|]), \quad k = 1, \ldots, n$$

where k refers to different neural response dimensions, such as sound, “feeling” which is a mixture of sensations (a composite feeling), and so on. Their product is a function of several complex variables and is the solution of the following equation.

$$\prod_k w_k(a_k z_k) = \prod_k b_k w_k(z_k)$$

The product of solutions of $w_k(a_k z_k) = b_k w_k(z_k)$ satisfies such an equation with the new b=☎b_k. Since the product of periodic functions of period 1 is also a periodic function of period one, the result of taking the product has the same form as the original function: a damping factor multiplied by a periodic function of period 1. If we multiply n solutions in the same variable z, in each of which b and W are allowed to be different, perhaps by adopting different forms for the periodic component, we obtain:

$$(b_1 b_2 \cdots b_n)^{[\log|z|/\log|a|]} W_1(z/a^{[\log|z|/\log|a|]}) W_2(z/a^{[\log|z|/\log|a|]}) \cdots$$

$$W_n(z/a^{[\log z/\log|a|]}) = b^{[\log|z|/\log|a|]} V(z/a^{[\log|z|/\log|a|]})$$

One then takes the Fourier transform of this solution.

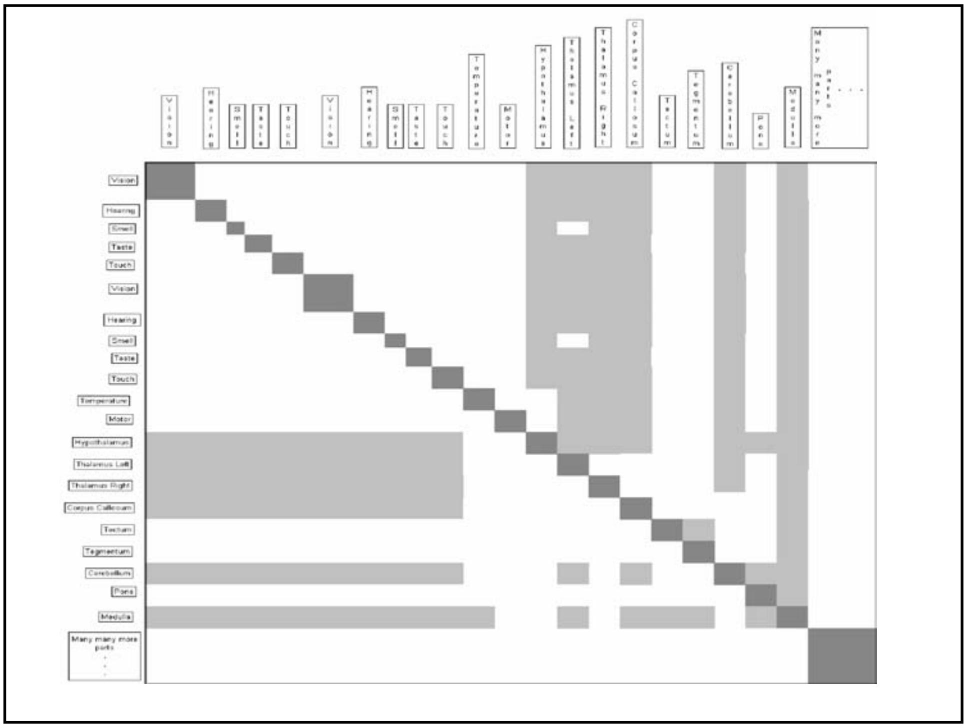
Graphing Complex Functions

Complex functions cannot be drawn as one does ordinary functions of three real variables because of their imaginary part. Nevertheless, one can make a plot of the modulus or absolute value of such a function. The density of linear combinations of Dirac-type functions or of approximations to them makes it possible to plot in the plane a version of our complex-valued solution.

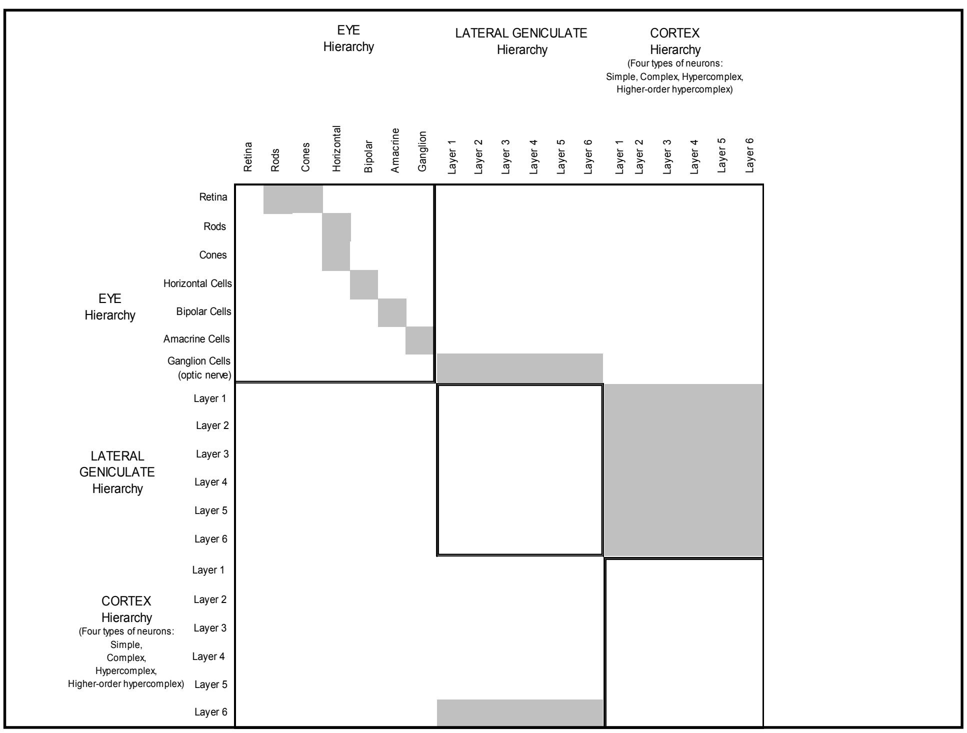
The Brain Hypermatrix

$$\tilde{W} = \begin{array}{c} \\ C_1 \\ C_2 \\ \cdots \\ C_k \\ C_{k+1} \\ C_{k+2} \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & \cdots & C_k & C_{k+1} & C_{k+2} \end{array} \\ \begin{bmatrix} W_{11} & 0 & \cdots & 0 & 0 & 0 \\ 0 & W_{22} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & W_{kk} & 0 & 0 \\ W_{k+1,1} & W_{k+1,2} & \cdots & W_{k+1,k} & W_{k+1,k+1} & W_{k+1,k+2} \\ W_{k+2,1} & W_{k+2,2} & \cdots & W_{k+2,k} & W_{k+2,k+1} & W_{k+2,k+2} \end{bmatrix} \end{array}$$

We need to raise this matrix to a sufficiently large power to connect all the parts that interact. The brain itself does this through feedback firings.



Vision
Hearing
Smell
Taste
Touch
Temperature
Motor
Hypothalamus
Thalamus Left
Thalamus Right
Corpus Callosum
Tectum
Tegmentum
Cerebellum
Pons
Medulla
Many many more parts



EYE Hierarchy
LATERAL GENICULATE Hierarchy
CORTEX Hierarchy
(Four types of neurons: Simple, Complex, Hypercomplex, Higher-order hypercomplex)
Retina
Rods
Cones
Horizontal
Bipolar
Amacrine
Ganglion
Layer 1
Layer 2
Layer 3
Layer 4
Layer 5
Layer 6
Horizontal Cells
Bipolar Cells
Amacrine Cells
Ganglion Cells (optic nerve)

- Since our solution is a product of two factors, the inverse transform can be obtained as the convolution of two functions, the inverse Fourier transform of each of which corresponds to just one of the factors.
- Now the inverse Fourier transform of $e^{-\beta u}$ is given by

$$\frac{\sqrt{(2/\pi)\beta}}{\beta^2+\xi^2}$$

Also because of the above discussion on Fourier series, we have

$$P(u)=\sum_{k=-\infty}^{\infty}\alpha_k e^{2\pi iku}$$

- whose inverse Fourier transform is:

$$\sum_{k=-\infty}^{\infty}\alpha_k\delta(\xi-2\pi k)$$

- Now the product of the transforms of the two functions is equal to the Fourier transform of the convolution of the two functions themselves which we just obtained by taking their individual inverse transforms.
- We have, to within a multiplicative constant:

$$\int_{-\infty}^{+\infty}\sum_{k=-\infty}^{\infty}\alpha_k\delta(\xi-2\pi k)\frac{\beta}{\beta^2+(\chi-\xi)^2}d\xi=\sum_{k=-\infty}^{\infty}\alpha_k\frac{\beta}{\beta^2+(x-2\pi k)^2}$$

- We have already mentioned that this solution is general and is applicable to phenomena requiring relative measurement through ratio scales. Consider the case where

$$P(u)=\cos u/2\pi=(1/2)(e^{iu/2\pi}+e^{-iu/2\pi})$$

- Bruce W. Knight adopts the same kind of expression for finding the frequency response to a small fluctuation and more generally using $e^{iu/2\pi}$ instead. The inverse Fourier transform of $w(u) = Ce^{-\beta u} \cos u / 2\pi, \beta > 0$ is given by:

$$c\frac{\beta}{\sqrt{2\pi}}\left[\frac{1}{\beta^2 + \left(\frac{1}{2\pi} + \xi\right)^2} + \frac{1}{\beta^2 + \left(\frac{1}{2\pi} - \xi\right)^2}\right]$$

- When the constants in the denominator are small relative to ξ we have c_1 / ξ^2 which we believe is why optics, gravitation (Newton) and electric (Coulomb) forces act according to inverse square laws. This is the same law of nature in which an object responding to a force field must decide to follow that law by comparing infinitesimal successive states through which it passes. If the stimulus is constant, the exponential factor in the general response solution given in the last chapter is constant, and the solution in this particular case would be periodic of period one. When the distance ξ is very small, the result varies inversely with the parameter β >0.

- The brain generally miniaturizes its perceptions into what may be regarded as a model of what happens outside. To control the environment there needs to be proportionality between the measurements represented in the miniaturized models that arise from the firings of our neurons, and the actual measurements in the real world. Thus our response to stimuli must satisfy the fundamental functional equation F(ax) = bF(x). In other words, our interpretation of a stimulus as registered by the firing of our neurons is proportional to what it would be if it were not filtered through the brain.

$w(z)=z^{\ln b/\ln a}\,P(\ln z/\ln a)$ whose space-time Fourier transform is a combination of Dirac distributions. Our solution of Fredholm's equation here is given as the Fourier transform,

$$f(\omega) = \int_{-\infty}^{+\infty} F(x)\, e^{-2\pi i\omega x}\, dx = Ce^{\beta\omega}\, P(\omega)$$

The response function $w(s)$ of the neuron in spontaneous activity results from solving the homogeneous Fredholm equation and is given by

$$w(t) \;=\; t^{\alpha}\, e^{g(t)} \;/ \int_{0}^{b} t^{\alpha}\, e^{g(t)}\; dt$$

for some choice of $g(t)$. Because finite linear combinations of the functions $\{t^{\alpha}\, e^{-\beta t},\ \alpha,\ \beta\ \geq\ 0\}$ are dense in the space of bounded continuous functions **C**$[0,b]$ we can approximate $t^{\alpha}\, e^{g(t)}$ by linear combinations of $t^{\alpha}\, e^{-\beta t}$ and hence we substitute $g(t)\;=\;-\beta t,\ \beta\ \geq\ 0$ in the eigenfunction $w(t)$.